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Gain adaptation in sliding mode control of robotic manipulators

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In this paper, a novel scheme is proposed to adapt the gains of a sliding mode controller (SMC) so that the problems faced in its practical implementations as a motion controller are overcome. A Lyapunov function is selected for the design of the SMC and an MIT rule is used for gain adaptation. The criterion that is minimized for gain adaptation is selected as the sum of the squares of the control signal and the sliding surface function. This novel approach is tested on a scara-type robot manipulator. The experimental results presented prove its efficacy.

1. Introduction

In many motion control applications, the system dynamics or the parameters may change with time. A powerful control technique for alleviating this problem is the use of variable structure system (VSS) theory with sliding mode control (Hung 1993). The technique is easy to use since only the bounds on the uncertain parameters need to be known (Utkin 1981).

VSSs with a sliding mode were first proposed in early 1950s, but it was not until the 1970s that sliding mode control became more popular. It nowadays enjoys a wide variety of application areas. The main reason for this popularity is the attractive properties of a sliding mode controller (SMC), such as the good control performance even in the case of nonlinear systems, the applicability to multiple-input multiple-output systems, the availability of design criteria for discrete-time systems, etc. The best property of the SMC is its robustness. Loosely speaking, a system with an SMC is insensitive to parameter changes or external disturbances.

The essential characteristic of a VSS is that the feedback signal is discontinuous, switching on one or more manifolds in state space. When the state crosses each discontinuity surface, the structure of the feedback system is altered. All motion in the neighbourhood of the manifold is directed towards the manifold and thus sliding motion occurs in which the system state repeatedly crosses the switching surface (Utkin 1981, Slotine and Li 1991).

The theory of VSSs with a sliding mode has been studied intensively by many researchers. Motion control, especially in robotics, has been an area that has attracted particular attention and numerous reports have appeared in the literature (Wijesoma 1990, Young 1993, Denker and Kaynak 1994, Tunay and Kaynak 1995, Ertugrul *et al.* 1996 a, b). A recent survey has been given by Hung (1993).

The classical SMC design, based on the selection of a Lyapunov function, results in a control input which is the sum of an equivalent control and a corrective control term (Ertugrul 1996 a). The equivalent control is the control that makes the derivative of the sliding surface function equal to zero. The corrective term, which is directly proportional to the sign of the sliding surface function, is used to compensate the deviations from the sliding surface.

In practical motion control applications, an SMC suffers mainly from two disadvantages. The first is the high-frequency oscillations of the controller output, termed 'chattering'. The second disadvantage is the difficulty involved in the calculation of what is known as the equivalent control. A thorough knowledge of the plant dynamics is required for this purpose (Ertugrul 1996 a). In the literature, some suggestions are made to overcome these problems. The most popular technique for the elimination of the chattering is the use of a saturation function, which was proposed by Slotine and Li (1991). To avoid the computational burden involved in the calculation of the equivalent control an estimation technique can be used (Ertugrul 1996 b). More recently, the use of 'intelligent' techniques based

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on fuzzy logic, neural networks, evolutionary computing and other techniques adapted from artificial intelligence have also been suggested (Erbatur *et al.* 1996, Jezernik *et al.* 1997, Ertugrul and Kaynak 1998).

An adaptive system is any physical system that has been designed with an adaptive viewpoint, and an adaptive controller is a controller that can modify its behaviour in response to changes in the dynamics of the process and the disturbances (Aström and Wittenmark 1989, Zhao and Utkin 1996). In this paper, a gain adaptation scheme is proposed which directly results in chattering-free control action. Additionally, an estimation scheme is proposed to compute the equivalent control.

The paper is organized as follows: the next section is devoted to the SMC approach and its implementation via the estimation of equivalent control. In § 3, a gain adaptation technique for the SMC is proposed, based on the well-known MIT rule (Aström and Wittenmark 1989). In § 4 the experimental studies carried out on a two-degrees-of-freedom direct-drive scara-type robot to appraise the performance of the proposed adaptation technique are described, and § 5 concludes the paper.

2. Variable structure systems

In the application of VSS theory to the control of nonlinear processes, it is argued that one only needs to drive the error to a 'switching' or 'sliding' surface, after which the system is in 'sliding mode' and will not be affected by any modelling uncertainties and/or disturbances (Utkin 1981, Hung 1993).

Intuitively, a VSS with a sliding mode is based on the argument that the control of first-order systems is much easier, even when they are nonlinear or uncertain, than the control of general *n*th-order systems (Slotine and Li 1991).

2.1. *The system (plant)*

Consider a nonlinear, multiple-input multiple-output system of the form

$$x_i^{(k_i)} = f_i(X) + \sum_{j=1}^{m} b_{ij} u_j, \tag{1}$$

where $x_i^{(k_i)}$ means the k_ith derivative of x_i. Also, the vector U of components u_j is the control input vector and the state X is composed of the x_i and their first $k_i - 1$ derivatives. Such systems are called square systems since they have as many control inputs as outputs x_i to be controlled. The system can be written in a more compact form by letting

$$X = [x_1 \ \dots \ x_m \quad \dot{x}_1 \ \dots \ \dot{x}_m \ \dots \ x_1^{(k_1-1)} \ \dots \ x_m^{(k_m-1)}]^{\mathrm{T}}, \tag{2}$$

$$U = \begin{bmatrix} u_1 & \dots & u_m \end{bmatrix}^{\mathrm{T}}. \tag{3}$$

Assuming X is $(n \times 1)$, the system equation becomes,

$$\dot{X}(t) = F(X) + B\,U(t), \tag{4}$$

where B is an $(n \times m)$ input gain matrix.

2.2. *Sliding surface*

For the system given in (4), the sliding surface that is represented by S is generally an $(m \times 1)$ matrix and it is selected (Ertugrul *et al.* 1996 a) as,

$$S(t) = GE(t) = G[X^{\mathrm{d}}(t) - X(t)] = \phi(t) - S_a(X), \tag{5}$$

where

$$\phi(t) = G\,X^{\mathrm{d}}(t), \quad S_a(X) = G\,X(t). \tag{6}$$

In the above, $\phi(t)$and $S_a(X)$ are the time- and state-dependent parts respectively, X^{d} represents the desired (reference) state vector which is selected in the form of X as in (2) and G is the $(m \times n)$ slope matrix of the sliding surface. The G matrix is generally selected such that the sliding surface function becomes

$$S_i = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda_i\right)^{k_i - 1} e_i, \tag{7}$$

where S_i means the ith element of the S vector, e_i is the error for $x_i (e_i = x_i^{\mathrm{d}} - x_i)$ and λ_i is a positive constant selected by the designer. Therefore, e_i goes to zero when S_i equals to zero.

The objective in an SMC is to force the system states on to the sliding surface. Once the states are on the sliding surface, the system errors converge to zero with error dynamics dictated by the matrix G.

2.3. *Sliding mode controller design*

The method described in this section is based on the Lyapunov approach. The control should be chosen such that the candidate Lyapunov function satisfies the Lyapunov stability criteria (Slotine and Li 1991). A Lyapunov function is selected as

$$V(S) = \frac{S^{\mathrm{T}} S}{2}. \tag{8}$$

It can be noted that this function is positive definite. $(V(S = 0) = 0$ and $V(S) > 0 \forall S \neq 0.)$ The aim is that the derivative of the Lyapunov function is negative definite. This can be maintained if one can assure that

$$\frac{\mathrm{d}V(S)}{\mathrm{d}t} = -S^{\mathrm{T}} D \,\mathrm{sgn}\,(S), \tag{9}$$

where D is an $(m \times m)$ positive definite diagonal gain matrix, and sgn (S) indicates a signum function applied to each element of S, that is

$$\mathrm{sgn}\,(S) = [\mathrm{sgn}\,(S_1) \quad \dots \quad \mathrm{sgn}\,(S_m)]^{\mathrm{T}}. \tag{10}$$

In the above, $\mathrm{sgn}\,(S_i)$ is defined as

$$\mathrm{sgn}\,(S_i) = \begin{cases} +1, & S_i > 0, \\ -1, & S_i < 0. \end{cases} \tag{11}$$

Taking the derivative of (8), and equating this to (9), one obtains

$$S^{\mathrm{T}} \frac{\mathrm{d}S}{\mathrm{d}t} = -S^{\mathrm{T}} D\, \mathrm{sgn}\,(S). \tag{12}$$

By taking the time derivative of (5) and using the plant equation,

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{\mathrm{d}\phi}{\mathrm{d}t} - \frac{\partial S_a}{\partial X}\frac{\mathrm{d}X}{\mathrm{d}t} = \frac{\mathrm{d}\phi}{\mathrm{d}t} - G[F(X) + BU] \tag{13}$$

is obtained. By putting (13) into (12), the control input signal turn out to be

$$U(t) = U_{\mathrm{eq}}(t) + U_{\mathrm{c}}(t), \tag{14}$$

where $U_{\mathrm{eq}}(t)$ is the equivalent control given by

$$U_{\mathrm{eq}}(t) = -(GB)^{-1}\left(GF(X) - \frac{\mathrm{d}\phi(t)}{\mathrm{d}t}\right) \tag{15}$$

and $U_{\mathrm{c}}(t)$ is the corrective control term given by

$$U_{\mathrm{c}}(t) = (GB)^{-1} D\, \mathrm{sgn}\,(S) = K\, \mathrm{sgn}\,(S). \tag{16}$$

2.4. *Chattering elimination*

The controller in (14) results in high-frequency oscillations in its output, causing a problem known as chattering. Chattering is undesirable because it can excite the high-frequency dynamics of the system. For its elimination, it is suggested that a shifted sigmoid function (Ertugrul *et al.* 1996 a) or a saturation function (Slotine and Li 1991) is used instead of the sign function. In the latter case, the corrective control term is given by

$$U_{\mathrm{c}}(t) = Kh(S), \tag{17}$$

where $h(S)$ is the saturation function defined as

$$h(S_j) = \begin{cases} S_j, & \text{if } -1 < S_j < 1, \\ \mathrm{sgn}\,(S_j), & \text{otherwise.} \end{cases} \tag{18}$$

2.5. *Estimation of the equivalent control*

As can be seen from (15), U_{eq} is a function of $F(X)$ and B and, if the knowledge of these matrices is very poor, then the equivalent control calculated will be too far off from the actual equivalent control. In the literature, a number of approaches are proposed for the estimation of U_{eq}, rather than calculating it. In this paper, an approach recently suggested in the literature (Ertugrul *et al.* 1996 b) is used, which is based on the fact that the equivalent control is actually the average of the total control (Utkin 1981). An averaging filter for calculation of the equivalent control can be designed as,

$$\tau \dot{\tilde{U}}_{\mathrm{eq}}(t) + \tilde{U}_{\mathrm{eq}}(t) = U(t). \tag{19}$$

It can also be written as

$$\tilde{U}_{\mathrm{eq}}(t) = \frac{1}{\tau p + 1} U(t), \tag{20}$$

where $\tilde{U}_{\mathrm{eq}}$ is an estimate for U_{eq} and $p = \mathrm{d}/\mathrm{d}t$. The average of the control is computed and fed back to calculate the control to be applied in the next control cycle. This method requires less knowledge about the system and thus alleviates some of the problems resulting from the uncertainties in the plant.

Equation (20) is actually a low-pass filter. The value of $1/\tau$ gives the cut-off frequency. The rationale behind designing a low-pass filter is that low frequencies determine the characteristics of the signal, and the high frequencies result from unmodelled dynamics.

One can now calculate the control as

$$U(t) = \tilde{U}_{\mathrm{eq}}(t) + (GB)^{-1} Dh(S), \tag{21}$$

where $\tilde{U}_{\mathrm{eq}}(t)$ is defined as in (20).

3. Adaptive sliding mode control

As is discussed above, conventional SMCs may result in an appreciable amount of chattering. The sign of the corrective control term U_{c}, which is added to the equivalent control, changes its sign frequently when S is around zero. If one takes this corrective term as constant, chattering is inevitable. This term should therefore be minimized when the states are on the sliding surface.

In this paper, an adaptation scheme based on the well-known MIT rule (gradient descent) is proposed to minimize the control effort and the sliding surface function. The criterion (cost function) which is to be minimized is chosen as

$$J = \tfrac{1}{2}(S^{\mathrm{T}} S + U_{\mathrm{c}}^{\mathrm{T}} U_{\mathrm{c}}). \tag{22}$$

The reason behind the selection of the cost function as in (22) can be stated as follows: minimizing the square of S results in a decrease in the error because S is a function of the error as defined in (5) and optimizing U_{c} reduces chattering.

To make J small, it is reasonable to change the parameters (weights) in the direction of its negative,

$$\frac{\mathrm{d}D_{ji}}{\mathrm{d}t} = -\gamma \frac{\partial J}{\partial D_{ji}}, \tag{23}$$

where, $j = 1, \dots, m$, $i = 1, \dots, m$, and D_{ji} stands for the entries of D.

The effect of the design parameter D on the performance of the system is shown in figure 1 for two-dimensional case. Curve 1 corresponds to the case when D is large. The system states reach the sliding line in a short time but overshoot it by a considerable amount. Curve 2 reflects the case with a small D parameter. Neither curve 1, nor curve 2 is very desirable. Curve 3 in the phase plane can be obtained via an adaptation algorithm.

The gradient descent as in (23) for D can be derived (by using the chain rule) as

$$\frac{\mathrm{d}D_{ji}}{\mathrm{d}t} = -\gamma \frac{\partial J}{\partial S_j}\frac{\partial S_j}{\partial D_{ji}} - \gamma \frac{\partial J}{\partial Uc_j}\frac{\partial Uc_j}{\partial D_{ji}}. \tag{24}$$

Using (22), the partial derivatives of the cost function can be calculated as

$$\frac{\partial J}{\partial S_j} = S_j, \quad \frac{\partial J}{\partial Uc_j} = Uc_j. \tag{25}$$

Using (16) and assigning the constants which result from the multiplication of elements G and B matrices as γ'_2, the partial derivative of the corrective term with respect to elements of D matrix is calculated as

$$\frac{\partial Uc_j}{\partial D_{ji}} = \gamma'_2 h(S_i). \tag{26}$$

The sliding surface function in (5) can be rewritten, using (21) and the integral of (4), as

$$S = G(X^{\mathrm{r}} - X) = GX^{\mathrm{r}} - G\int_{t_0}^{t} \times \{F(X) + B[\tilde{U}_{\mathrm{eq}} + (GB)^{-1}Dh(S)]\}\,\mathrm{d}\xi \tag{27}$$

Taking the partial derivative of (27) with respect to D_{ji} and assigning the constants to γ'_1which are obtained by multiplication of elements of G and B,

$$\frac{\partial S_j}{\partial D_{ji}} = -\gamma'_1 \int_{t_0}^{t} h(S_i(\xi))\,\mathrm{d}\xi. \tag{28}$$

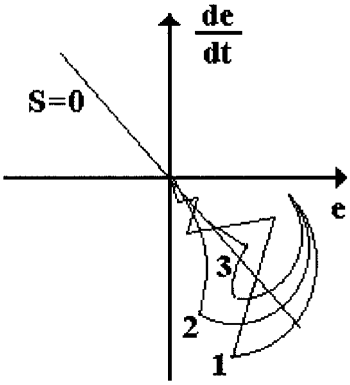

Figure 1. The motion on the sliding line for different D values. This also shows the effect of the D adaptation.

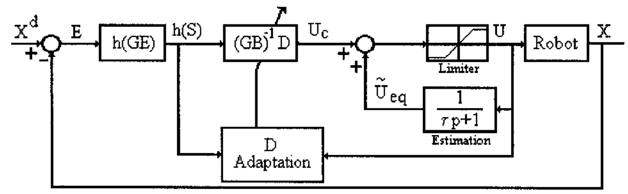

Figure 2. The structure of the overall system with the proposed controller.

The last form of D adaptation is obtained as

$$\frac{\mathrm{d}D_{ji}}{\mathrm{d}t} = -\gamma''_2 Uc_j h(S_i) + \gamma''_1 S_j \int_{t_o}^{t} h(S_i(\xi))\,\mathrm{d}\xi. \tag{29}$$

The adaptation rule in (29) can be rewritten for discrete time systems in recursive form as

$$D_{ji}(k+1) = D_{ji}(k) - \gamma_2 Uc_j(k)h(S_i(k)) + \gamma_1 S_j(k)\left(\sum_{l=1}^{k} h(S_i(j))T\mathrm{s}\right), \tag{30}$$

where Ts is the sampling time of the discrete system, and $\gamma, \gamma_1, \gamma_2, \gamma'_1, \gamma'_2, \gamma''_1$, and γ''_2 are constants. γ_1 and γ_2 are also known as the adaptation rates.

In the controller design, D is selected as a diagonal gain matrix. Therefore, there is no need to adapt the gains which are not on the diagonal:

$$D_{ji} = \begin{cases} D_{ii}, & \text{if } j = i, \\ 0, & \text{otherwise.} \end{cases} \tag{31}$$

Because of the definition of D in the Lyapunov function of (9), D_{ii} has to be positive. Therefore, the calculated D_{ii} is passed through a minimum limiter as

$$D_{ii} = \max[0.5, D_{ii}]. \tag{32}$$

The lower value of D_{ii} is selected as 0.5 instead of zero in order to suppress the errors arising from the estimation of the equivalent control.

Combining (31) and (32), the last form is obtained as

$$D_{ji} = \begin{cases} D_{ii}, & (i = j) \text{ and } (D_{ii} > 0.5), \\ 0.5, & (i = j) \text{ and } (D_{ii} \leqslant 0.5), \\ 0, & i \neq j \end{cases} \tag{33}$$

The structure of the overall system with the proposed controller is presented in figure 2.

4. Robotics application

In order to study the performance of the proposed controller, extensive implementation studies are carried out on a two-degrees-of-freedom direct-drive scara-type experimental manipulator shown in figure 3, which is manufactured by Integrated Motion Corporation (1992).

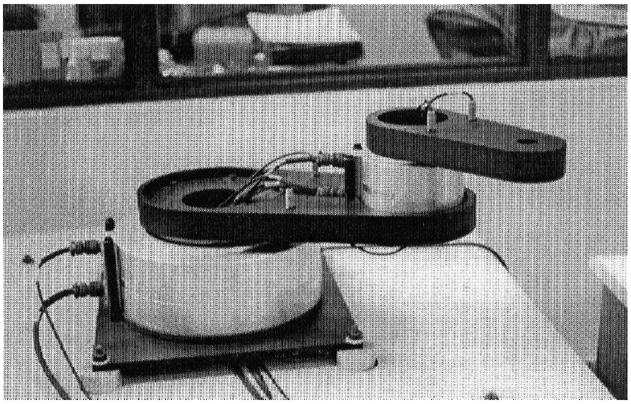

Figure 3. The experimental direct-drive Scara robot.

4.1. *Robot dynamics*

The robot model is written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + f_C = \tau, \tag{34}$$

where $M(q)$ is the (2×2) inertia matrix, $C(q,\dot{q})$ are the (2×2) Coriolis terms, τ is the (2×1) torque vector, $q, \dot{q}, \ddot{q}$ are the (2×1) position, velocity and acceleration vectors respectively, f_C is the (2×1) Coloumb frictional force, $q = [\theta_1 \quad \theta_2]^T$ and θ_i are joint angles $(i = 1,2)$.

The details of the dynamics can be found in the guide by the Integrated Motion Corporation (1992). The model in (34) can be written in the state-space form representation as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -M^{-1}(Cx_2 + f_C) \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} u, \tag{35}$$

where

$$[x_1 \quad x_2]^T = [q \quad \dot{q}]^T = [\theta_1 \quad \theta_2 \quad \dot{\theta}_1 \quad \dot{\theta}_2]$$

and

$$u = \tau.$$

Equation (35) is in the form of (4), and the proposed method can be applied.

4.2. *The experimental set-up*

The control workstation has an open architecture, enabling the modifications of the control algorithm. The latter can be written and complied in C-language in a personal computer equipped with a 80486 central processing unit. The compiled form of the proposed control algorithm is downloaded to a DSP-based servo-controller. A TMS320C30 DSP is used which is a floating-point DSP with a 32 bit architecture. Necessary torques to track a desired trajectory are computed by software and written to DACs of the board. The motor driver, functionally, converts the complex variable-reluctance motor into a system that behaves like a high-torque low-velocity dc motor. It also amplifies the controller output to a level that is capable of driving the direct-drive motors. The only available feedback signals are the angular positions which are measured by encoders with 153 600 counts per actuator revolution. The angular velocities are computed by differentiating the measured positions. The system parameters are set to work in torque mode. The architecture of the controller is presented in figure 4.

4.3. *Experimental results*

The experimental results are presented from figures 5 to 12, where the solid and the broken curves are related to the base and elbow link respectively. The desired state trajectories used are depicted in figure 5. The angular errors are presented in figure 6. It should be pointed out that the initial position errors are deliberately introduced to see the system behaviour (figures 7 and 8) when the system is not on the sliding surface. Theoretically, when the states reach to the origin on the phase plane, they should stay there. However, in DSP-based discrete-time applications, because the system is open loop for the duration of the sampling time, they deviate from origin. When the control for the next sampling time is applied, the states again move towards the origin. In

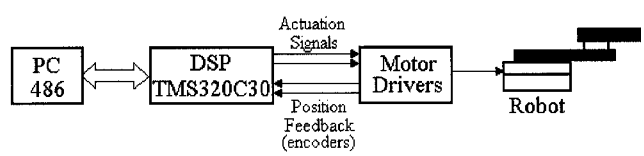

Figure 4. The controller structure.

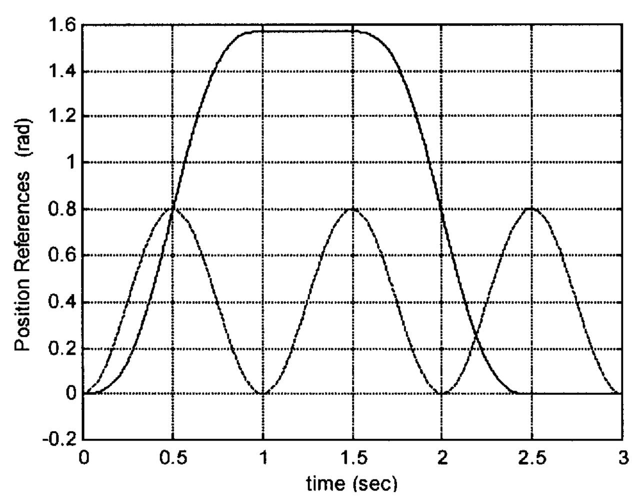

Figure 5. Reference angular position trajectories for the base and the elbow links: (——), base link; (- - - -), elbow link.

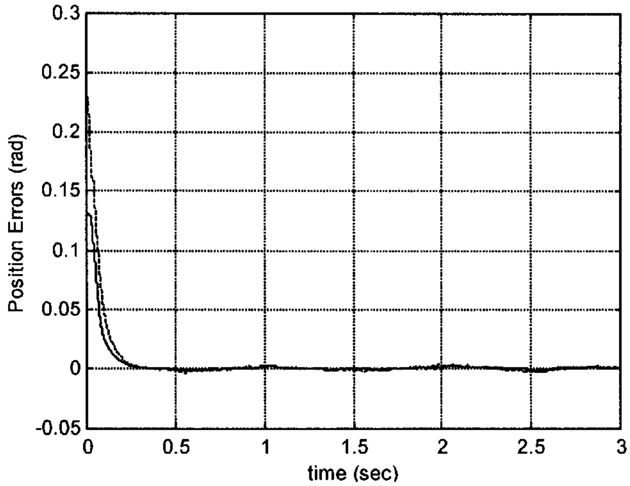

Figure 6. The angular errors

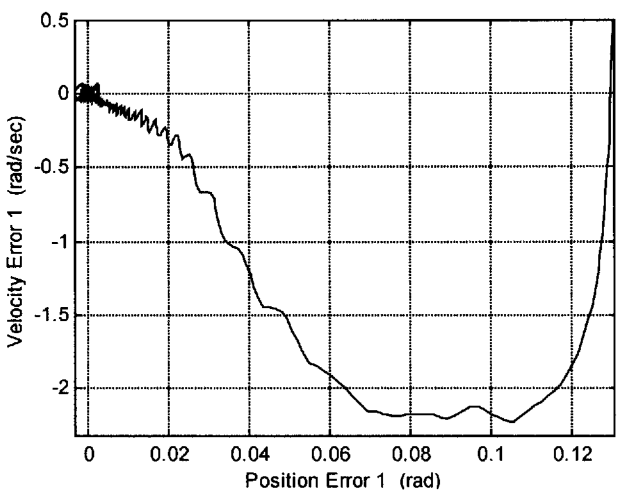

Figure 7. The motion on phase plane 1.

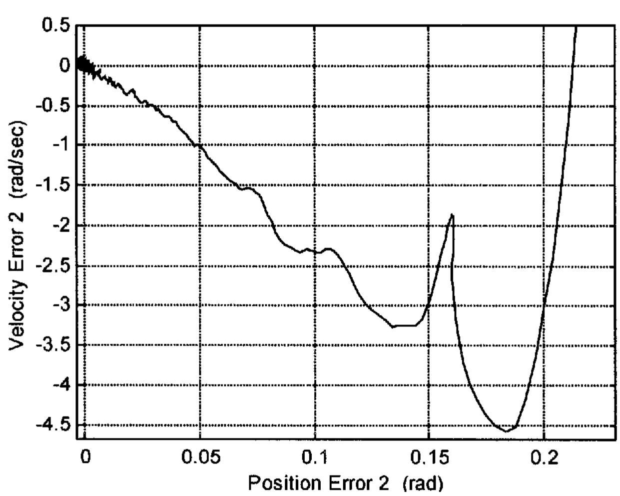

Figure 8. The motion on phase plane 2.

other words, the proposed controller establishes a stable domain of attraction around the origin. The control signals that are applied to the robot are presented in figure 9. They show some oscillations at the beginning owing to the high-gain selection and then the oscillations are eliminated owing to the D adaptation. The equivalent control that is estimated by a low-pass filter is presented in figure 10. As is expected, it is a continuous signal being the average of the control signal. The corrective term of the SMC is presented in figure 11. The adapted parameter D is presented in figure 12. The excessive activity seen in the control signal at the beginning indicates that the initial D values are high. The proposed D-adaptation algorithm successfully brings the D values to a level such that ringing is eliminated. It is seen that a gain-adaptive SMC results in very good trajectory following performance.

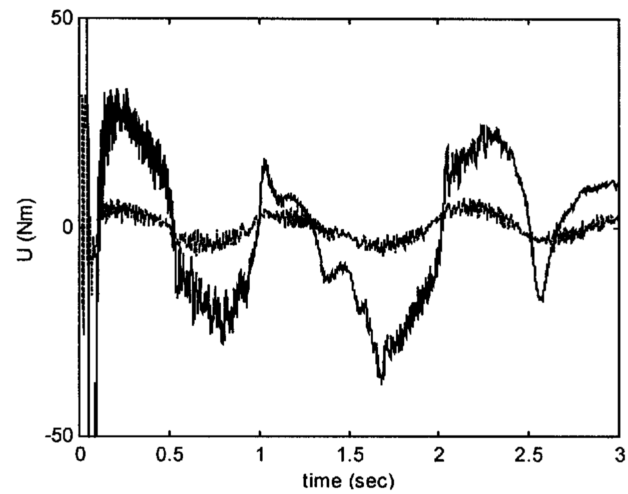

Figure 9. The controller outputs.

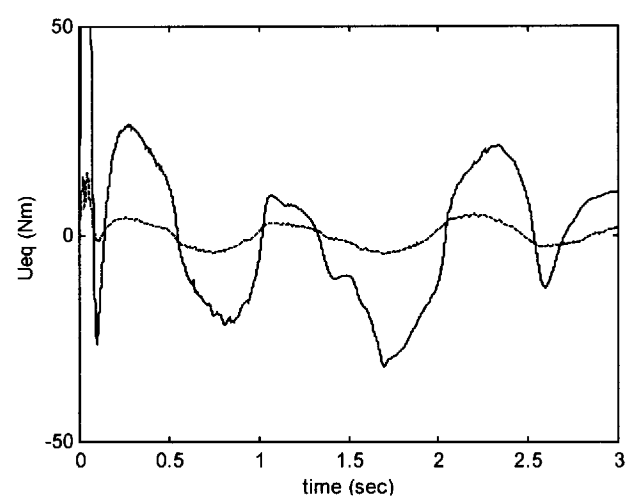

Figure 10. The equivalent controls.

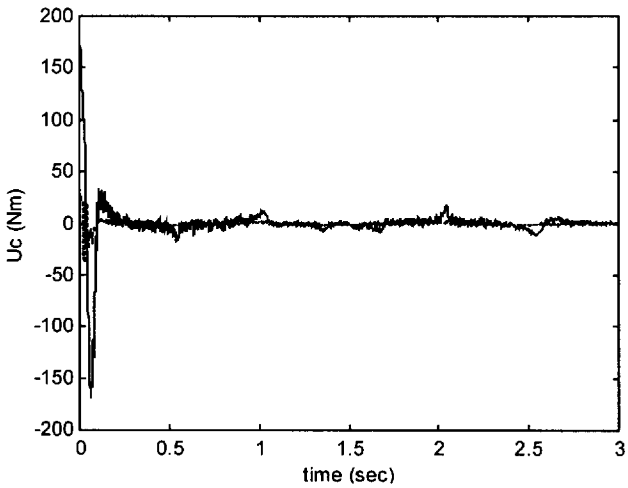

Figure 11. The corrective controls.

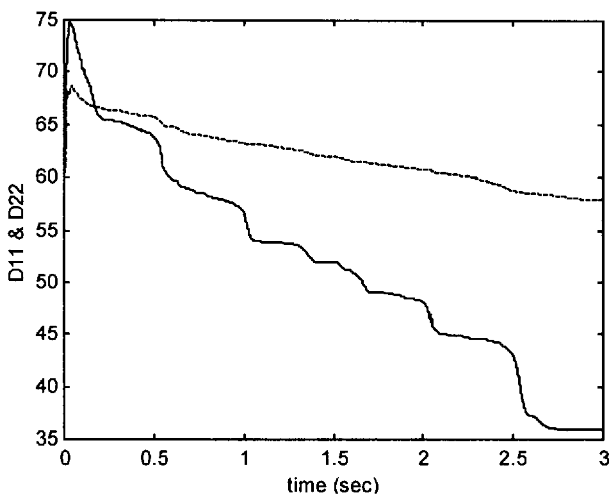

Figure 12. The adapted parameter *D*.

Extensive implementation studies are carried out with both constant and adapted gains. In both cases, the manipulator follows the desired trajectory, but the controller outputs for constant gains exhibit persistent ringing. Regarding the trajectory following performance, the error for the adaptive case is appreciably less than that observed with the classical SMC.

It is to be noted that the equivalent control is not calculated as in (15), but the estimation technique given by (20) is used in both cases. The *G* and *D matrices* are selected as

$$G = \begin{bmatrix} 20 & 0 & 1 & 0 \\ 0 & 20 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}.$$

The essential tuning parameters in adaptation are the γ_i in (29). When selecting a value for it, there are two criteria: adaptation capability and stability. While a low value causes low adaptation capability, a high value may cause instability owing to an excitation of the high-frequency dynamics. Consequently, a sufficiently large value that does not make the system unstable should be chosen. The experimental results presented are obtained by $\gamma_1 = 1.0$ and $\gamma_2 = 0.15$.

5. Conclusions

In this paper, a scheme using the MIT rule is applied for the purpose of adapting the gains of a classical SMC which is designed by selecting a Lyapunov function. The goal is to eliminate chattering and to reduce the tracking error. Therefore, the cost function to be minimized by the MIT rule is selected as the sum of squares of the control signal and the sliding surface function.

The SMC design based on the selection of a Lyapunov function (as described in this paper) results in a control signal that is made up of an equivalent control term plus a corrective term. The latter is necessary when the system states deviate from the sliding surface. It pushes them back towards the surface. The major reason why chattering occurs in the classical SMC is the unnecessarily large gain used in this term. Consequently, keeping it to a minimum when the states are in the immediate vicinity of the sliding surface will minimize chattering. This can be achieved by minimizing the multiplicative gain *D*. This is why an adaptation scheme is proposed in this paper. The corrective term can be minimized but actually not be made zero because the system may deviate from the sliding surface for such reasons as external disturbances, sharp changes in the reference signal, and errors caused by the estimation of the equivalent control. A minimum value of the corrective term can be achieved by the use of a limiter to counteract the cases when the adaptive scheme results in an excessively small value, zero or even negative values.

The experimental results presented in this paper indicate that the suggested approach has considerable advantages compared with the conventional approach and is capable of achieving a good chatter-free trajectory following performance without an exact knowledge of the plant parameters. These characteristics make it a promising approach for motion control applications.

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