Simulated and Experimental Study of Antilock Braking System Using Grey Sliding Mode Control

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***Abstract*— Antilock Braking System (ABS) exhibits strongly nonlinear and uncertain characteristics. To overcome these difficulties, robust control methods should be employed. In this paper, a grey sliding mode controller is proposed to track the reference wheel slip. The concept of grey system theory, which has a certain prediction capability, offers an alternative approach to conventional control methods. The proposed controller anticipates the upcoming values of wheel slip, and takes the necessary action to keep wheel slip at the desired value. The control algorithm is applied to a quarter vehicle model, and it is verified through simulations indicating fast convergence and good performance of the designed controller. Simulated results are validated on real time applications using a laboratory experimental setup.**

I. INTRODUCTION

ABS is an electronically controlled system that helps the driver to maintain control of the vehicle during emergency braking while preventing the wheels to lock up. Furthermore, by keeping brake pressure just below the point of causing a wheel to lock, ABS ensures that maximum braking power is used to stop the vehicle, and minimum possible stopping distance is achieved.

During accelerating or braking, the generated friction forces are proportional to the normal load of the vehicle. The coefficient of this proportion is called road adhesion coefficient and it is denoted by μ. Studies show that μ is a nonlinear function of wheel slip, λ [1]. The typical μ-λ curve is obtained from the data of numerous experiments. Most of the ABS controllers are expected to keep the vehicle slip at a particular level, where the corresponding friction force (i.e. road adhesion coefficient) reaches its maximum value. Zanten states in [2] that the wheel slip should be kept between 0.08 and 0.3 to achieve optimal performance. Furthermore, some research papers show that the reference wheel slip does not have to be a constant value. In [3], the reference wheel slip is considered as a nonlinear function of some physical variables including the velocity of the vehicle.

Although many attempts have been made over the decades, an accurate mathematical model of ABS has not been obtained yet. One of the shortcomings is that the controller must operate at an unstable equilibrium point in order to get the optimal performance. A small perturbation of the controller input may result in a drastic change in the output. Furthermore in today's technology, there are no affordable sensors which can accurately identify the road surface, and make this data available to ABS controller. Regarding the fact that the system parameters highly depend on the road conditions and vary over a wide range, the performance of ABS may not always be satisfactory. Moreover, sensor signals exhibit usually highly uncertain and noisy characteristics [4].

Because of the highly nonlinear and uncertain structure of ABS, many difficulties arise in the design of a wheel slip regulating controller. Sliding mode control is a preferable option, as it guarantees the robustness of the system for changing working conditions. The stability requirements for switching surface are described in [5]. In [6] and [7], it is assumed that the optimal value of wheel slip, which will result in maximum braking torque, is known. Drakunov [8] employs sliding mode to achieve the maximum value of friction force without the priori knowledge of optimum slip value. Kachroo and Tomizuka proposed a Sliding Mode Controller (SMC) in [9] that can maintain the wheel slip at any desired value. Unsal [10] proposed a sliding mode observer to track the reference wheel slip, and a PI-like controller is used to reduce the chattering problem.

This paper proposes a SMC and a Grey Sliding Mode Controller (GSMC) for tracking a reference wheel slip. Due to highly nonlinear and uncertain characteristics of ABS, a grey predictor is employed to anticipate the future outputs of the system using current data available. Grey predictor estimates the forthcoming value of wheel slip, and SMC takes the necessary action to maintain wheel slip at the desired value. To investigate the performance of the proposed controller, the reference wheel slip is considered both a constant value and a nonlinear function of the vehicle velocity. In the next section, a laboratory setup of an ABS is described and its dynamic equations are derived. SMC and grey predictor are developed in Section 3 and in Section 4, respectively. Simulation and experimental real time results are provided and compared in Section 5. Section 6 makes some concluding remarks.

II. SYSTEM DESCRIPTION

The laboratory setup of ABS consists of two rolling wheels. The lower wheel imitates of relative road motion. The upper wheel mounted to the balance lever animates the wheel of the vehicle. While two rotary encoders are installed on both of the wheels to measure the angular velocities, an additional one is used to identify the angular position of the

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TABLE I

SYSTEM PARAMETERS

ω_1	Angular velocity of the upper wheel
ω_2	Angular velocity of the lower wheel
T_B	Braking torque
r_1	Radius of the upper wheel
r_2	Radius of the lower wheel
J_1	Moment of inertia of the upper wheel
J_2	Moment of inertia of the lower wheel
d_1	Viscous friction coefficient of the upper wheel
d_2	Viscous friction coefficient of the lower wheel
F_n	Total normal load
μ	Road adhesion coefficient
λ	Wheel slip
λ_R	Reference slip
F_t	Road friction force
M_{10}	Static friction of the upper wheel
M_{20}	Static friction of the lower wheel
M_g	Moment of gravity acting on balance lever

balance lever. In order to accelerate the lower wheel, a large flat DC motor is coupled on it. The upper wheel is equipped with a disc brake system that is driven by a small DC motor. The velocity of the car is defined to be equivalent to angular velocity of the lower wheel multiplied by the radius of this wheel. The angular velocity of the wheel is defined to be equivalent to the angular velocity of the upper wheel [11].

The free body diagram of the quarter vehicle model describing longitudinal motion of the vehicle and angular motion of the wheel under braking is presented in Fig. 1 [11]. Although the model is quite simple, it preserves the fundamental characteristics of an actual system. In deriving the dynamic equations of the system, several assumptions are made. First, only longitudinal dynamics of the vehicle is considered. The lateral and vertical motions are neglected. Furthermore, it is assumed that there is no interaction between the four wheels of the vehicle.

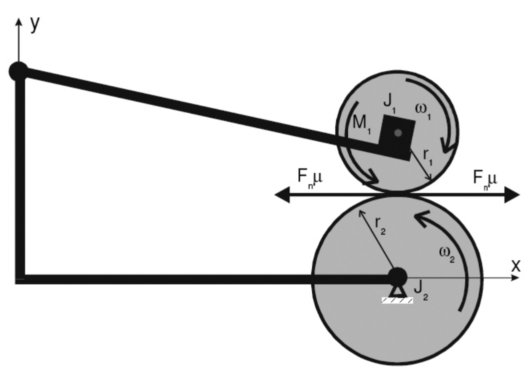

Fig. 1. Schematic view of experimental setup

Regarding the model, there are three torques acting on the upper wheel: Braking torque, friction torque in the upper bearing and the friction torque among the wheels. Similarly, two torques are acting on the lower wheel: The friction torque in the lower bearing and the friction torque between these wheels.

During deceleration, a braking torque is applied to the upper wheel, which causes wheel speed to decrease. According to Newton's second law, the equation of the motion of the system can be written as:

$$J_1\dot{\omega}_1 = F_t r_1 - (d_1\omega_1 + M_{10} + T_B) \tag{1}$$

$$J_2\dot{\omega}_2 = -(F_t r_2 + d_2\omega_2 + M_{20}) \tag{2}$$

F_t in (1) and (2) stands for the road friction force which is given by Coulomb Law:

$$F_t = \mu(\lambda)F_n \tag{3}$$

F_n is calculated by the following equation:

$$F_n = \frac{d_1\omega_1 + M_{10} + T_B + M_g}{L(\sin\phi - \mu(\lambda)\cos\phi)} \tag{4}$$

where L is the distance between the contact point of the wheels and the rotational axis of the balance lever and ϕ is the angle between the normal in the contact point and the line L.

Under normal operating conditions, the rotational velocity of the wheel would match the forward velocity of the car. When the brakes are applied, braking forces are generated at the interface between the wheel and road surface, which causes the wheel speed to decrease. As the force at the wheel increases, slippage will occur between the tire and the road surface. The wheel speed will tend to be lower than vehicle speed. The parameter used to specify this difference in these velocities is called wheel slip (λ), and it is defined as:

$$\lambda = \frac{r_2\omega_2 - r_1\omega_1}{r_2\omega_2} \tag{5}$$

While a wheel slip of 0 indicates that the wheel velocity and the vehicle velocity are the same, a ratio of 1 indicates that the tire is not rotating and the wheels are skidding on the road surface, i.e., the vehicle is no longer steerable.

The road adhesion coefficient is a nonlinear function of some physical variables including wheel slip and it can be approximated by the following formula [11]:

$$\mu(\lambda) = \frac{c_4\lambda^p}{a + \lambda^p} + c_3\lambda^3 + c_2\lambda^2 + c_1\lambda \tag{6}$$

The numerical values used in this study are:

$r_1 = 0.0995(m)$
$r_2 = 0.0990(m)$
$\phi = 65.61(°)$
$L = 0.37(m)$
$J_1 = 0.00753(kgm^2)$
$J_2 = 0.0256(kgm^2)$
$d_1 = 0.00011874(kgm^2/s)$
$d_2 = 0.00021468(kgm^2/s)$
$M_{10} = 0.0032(Nm)$
$M_{20} = 0.0925(Nm)$
$c_1 = -0.04240011450454$,
$c_2 = 0.00000000029375$,
$c_3 = 0.03508217905067$,
$c_4 = 0.40662691102315$,
$a = 0.00025724985785$, and
$p = 2.09945271667129$.

III. SLIDING MODE CONTROLLER (SMC)

Sliding mode control is a simple form of robust control, and it provides an effective method to control nonlinear plants. In sliding mode control, systems described by n^{th}-order differential equations are replaced by corresponding systems described by 1^{st}-order differential equation [12].

In this study, SMC is used to track reference wheel slip. The switching function, s, is defined as [13]:

$$s = \lambda - \lambda_R \tag{7}$$

The sliding motion occurs when the state $(\lambda_R, \dot{\lambda_R})$ reaches the sliding surface (a point in this case) defined by $s = 0$. The control that keeps the state on the sliding surface is called the equivalent control. In this study, it is called equivalent control brake torque, $T_{b,eq}$.

Since motion in the sliding mode

$$\dot{s} = 0 \tag{8}$$

Differentiating (6) and substituting into (7) results in:

$$\dot{\lambda} = \dot{\lambda}_R \tag{9}$$

Differentiating (5) gives:

$$\dot{\lambda} = \frac{r_1}{w_2 r_2 J_1}(-F_t r_1 + d_1 w_1 + M_{10} + T_B) - \frac{w_1 r_1}{w_2^2 r_2 J_2}(F_t r_2 + d_2 w_2 + M_{20}) \tag{10}$$

If it is assumed that the reference wheel speed is constant,

$$\dot{\lambda} = 0 \tag{11}$$

Substituting (10) into (9) gives:

$$0 = \frac{r_1}{w_2 r_2 J_1}(-F_t r_1 + d_1 w_1 + M_{10} + T_B) - \frac{w_1 r_1}{w_2^2 r_2 J_2}(F_t r_2 + d_2 w_2 + M_{20}) \tag{12}$$

Solving for the equivalent control brake torque, $T_{b,eq}$, gives:

$$T_{B,eq} = F_t r_1 - d_1 w_1 - M_{10} - \frac{w_1 J_1}{w_2 J_2}(F_t r_2 + d_2 w_2 + M_{20}) \tag{13}$$

If the system state (λ, λ_R) is not on the sliding surface, an additional control term called hitting control brake torque $T_{b,h}$, should be added to the overall brake torque control signal. When the system state is on the sliding surface, the hitting control has no effect. The hitting control brake torque $T_{b,h}$, is determined by the following reaching condition where η_s is a strictly positive design parameter.

$$s\dot{s} \leq -\eta_s |s| \tag{14}$$

Using (8) and (11), (14) can be rewritten as:

$$s\dot{\lambda} \leq -\eta_s |s| \tag{15}$$

Substitution of (10) into (15) results:

$$\frac{s r_1}{w_2 r_2 J_1}(-F_t r_1 + d_1 w_1 + M_{10} + (T_{B,eq} - T_{B,h} sgn(s)) - \frac{s w_1 r_1}{w_2^2 r_2 J_2}(F_t r_2 + d_2 w_2 + M_{20}) \leq -\eta |s| \tag{16}$$

Solving for the hitting control brake torque, $T_{B,h}$, results:

$$T_{B,h} \geq \frac{w_2 r_2 J_1}{w_1}\eta \tag{17}$$

The overall brake torque control is assumed to have the form

$$T_b = T_{b,eq} - T_{b,h} f(s) \tag{18}$$

To deal with the chattering problem, which usually occurs because of the discontinuous control input and high speed switching, the discontinuous switching function is replaced by the following continuous one where $\delta \geq 0$ [14]:

$$f(s) = \frac{s}{|s| + \delta} \tag{19}$$

IV. GREY SLIDING MODE CONTROLLER (GSMC)

Grey system theory is an interdisciplinary scientific area that was first introduced by Professor Deng Ju-Long from China in 1982. Prof. Deng published the first research paper, titled "Control Problems of Grey Systems", in the area of grey systems in the international journal, "Systems and Control Letters" [15]. The theory has since then become quite popular with its ability to deal with the systems that have partially unknown parameters. It is therefore a good candidate to real-time control systems [16]. In this paper, a grey predictor is used to forecast the angular velocity of the wheels and the linear velocity of the vehicle for using in SMC.

During the last two decades, the grey system theory has been developed rapidly and caught the attention of many researchers. It has been widely and successfully applied to various systems such as social, economic, financial, scientific and technological, agricultural, industrial, transportation, mechanical, meteorological, ecological, hydrological, geological, medical, military, etc., systems. Some early milestones are as follows: a grey prediction controller, combined with a conventional controller for an unknown system was proposed by Cheng in 1986 [17]. In 1994, Huang proposed the basic structure of grey prediction fuzzy model to control robotic motion and an inverted pendulum (which is a classical control problem) [18], [19]. In these studies and the others, it has been seen that grey system theory based approaches can achieve good performance characteristics when applied to real-time systems, since grey predictors adapt their parameters to new conditions as new outputs become available. Because of this reason, grey controllers are more robust with respect to noise, lack of modeling information, and to other disturbances when compared to conventional controllers.

GM(1,1) type of grey model is most widely used in the literature, pronounced as "Grey Model First Order One Variable". This model is a time series forecasting model.

The differential equations of the GM(1,1) model have time-varying coefficients. In other words, the model is renewed as the new data become available to the prediction model.

In order to smooth the randomness, the primitive data obtained from the system to form the GM(1,1) is subjected to an operator, named Accumulating Generation Operation (AGO) [20]. The differential equation (i.e. GM(1,1)) thus evolved is solved to obtain the n-step ahead predicted value of the system. Finally, using the predicted value, Inverse Accumulating Generation Operation (IAGO) is applied to find the predicted values of original data.

Consider a single input and single output system. Assume that the time sequence $X^{(0)}$ represents the outputs of the system:

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\right), n \geq 4 \tag{20}$$

where $X^{(0)}$ is a non-negative sequence and n is the sample size of the data. When this sequence is subjected to the AGO, the following sequence $X^{(1)}$ is obtained. It is obvious that $X^{(1)}$ is monotone increasing.

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)\right), n \geq 4 \tag{21}$$

where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, 3....., n \tag{22}$$

The generated mean sequence $Z^{(1)}$ of $X^{(1)}$ is defined as:

$$Z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), ..., z^{(1)}(n)\right) \tag{23}$$

where $z^{(1)}(k)$ is the mean value of adjacent data, i.e.

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), k = 2, 3, ..., n \tag{24}$$

The least square estimate sequence of the grey difference equation of GM(1,1) is defined as follows [20]:

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{25}$$

The whitening equation is therefore as follows:

$$\frac{dx^1(t)}{dt} + ax^{(1)}(t) = b \tag{26}$$

In above, $[a, b]^T$ is a sequence of parameters that can be found as follows:

$$[a, b]^T = (B^T B)^{-1} B^T Y \tag{27}$$

where

$$Y = \left[x^{(0)}(2), x^{(0)}(3), ..., x^{(0)}(n)\right]^T \tag{28}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ . & . \\ . & . \\ . & . \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{29}$$

According to (26), the solution of $x^{(1)}(t)$ at time k:

$$x_p^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a} \tag{30}$$

To obtain the predicted value of the primitive data at time (k+1), the IAGO is used to establish the following grey model.

$$x_p^{(0)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak}(1 - e^a) \tag{31}$$

and the predicted value of the primitive data at time $(k + H)$:

$$x_p^{(0)}(k+H) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-a(k+H-1)}(1 - e^a) \tag{32}$$

The parameter $(-a)$ in the GM(1,1) model is called "development coefficient" which reflects the development states of $X_p^{(1)}$ and $X_p^{(0)}$. The parameter b is called "grey action quantity" which reflects changes contained in the data because of being derived from the background values [21].

To integrate grey prediction into the proposed SMC, a new time-varying grey sliding surface $s(\lambda; t)$ is defined as [22]:

$$s(\lambda, t) = (e + e_p) \tag{33}$$

where e is the system error, $e_p = \lambda_R - \lambda_p$ is a value predicted by GM(1,1) model and λ_p is the predicted value of wheel slip. If the Lyapunov candidate function V_L is defined as:

$$V_L = \frac{1}{2}\left(s(\lambda, t)\right)^2 \tag{34}$$

then it is guaranteed that the tracking error of GSMC will be less than the one of conventional SMC [22]. The structure of the proposed GSMC is shown in Fig 2.

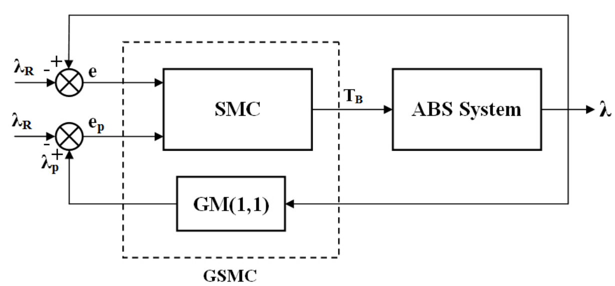

Fig. 2. Structure of GSMC

V. SIMULATED AND EXPERIMENTAL RESULTS

To investigate the performance of the proposed controllers, a number of computer simulated dynamic responses are obtained. Furthermore, the simulated designs are built and used in the real time experiments on the dynamic test stand. All figures below show simulation and experimental results for a car with initial longitudinal velocity of V=70km/h maneuvering on a straight line. The reference wheel slip is first taken to be constant ($\lambda = 0.2$), then it is considered as a function of vehicle velocity. All tests are run for a 1ms sampling period.

A. Simulated Results

Fig. 3 and Fig. 4 illustrate system responses of SMC and SMC coupled to a grey predictor for a constant reference wheel slip of $\lambda = 0.2$. The step size of GSMC is considered as a constant value which is 20. In order to obtain more realistic results, band-limited white noise is added to the system at slip measurements. The numerical values of noise power, the design parameters η and δ are selected as $1x10^{-5}$, 1.2 and 0.2, respectively. Although both controllers posses steady state errors, GSMC exhibits better performance when compared to conventional SMC like mentioned in Chapter IV. Both controllers are capable of stopping the car in approximately 1.3 seconds. The stopping distance for both controllers are similar.

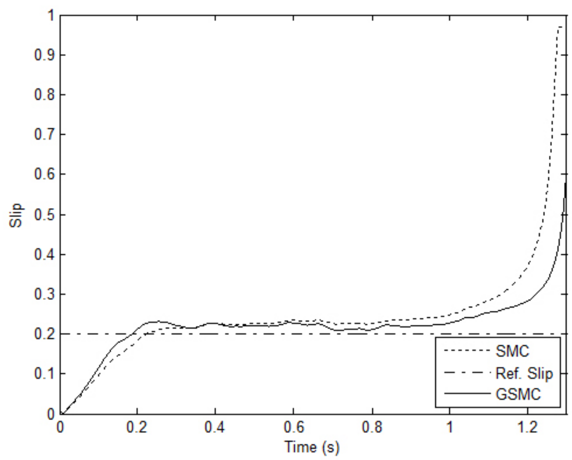

Fig. 3. Wheel slip of SMC and GSMC for a constant reference

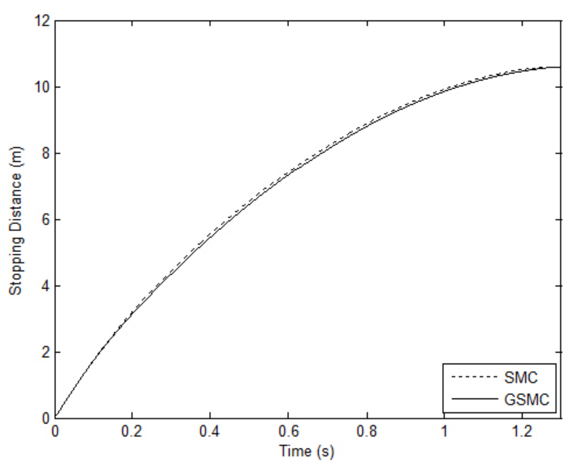

Fig. 4. Stopping distance of SMC and GSMC for a constant reference

Fig. 5 and Fig. 6 show simulation results of both controllers for velocity dependent reference wheel slip using the formula presented in [3]. Regarding the simulation results, it can be inferred that SMC and GSMC can track the reference wheel slip satisfactorily. Similar to the previous case, the steady state error in GSMC is less than conventional SMC, and the stopping distance values are approximately the same. The stopping time of the car is approximately 1.15 seconds, which is slightly better than previous case.

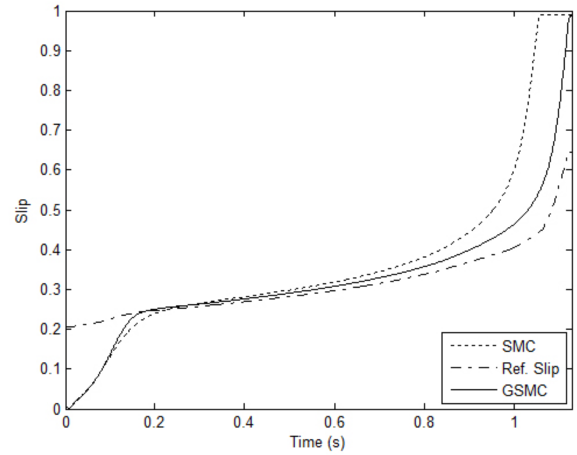

Fig. 5. Wheel slip of SMC and GSMC for a constant reference

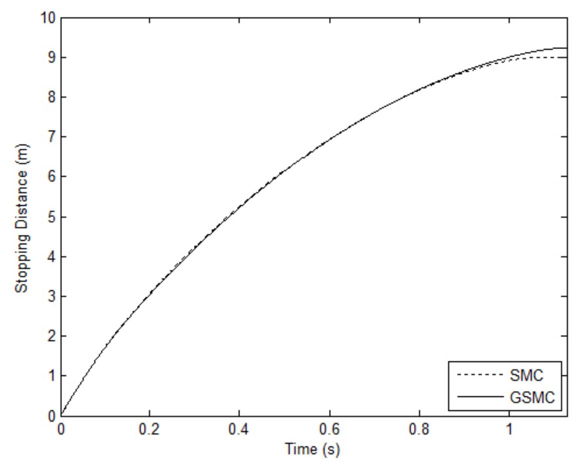

Fig. 6. Stopping distance of SMC and GSMC for velocity dependent reference

B. Experimental Results

For the experiments, the ABS setup of Inteco Ltd. is used [11]. To imitate the behaviour of the vehicle during braking on a dry and straight road, the wheel is accelerated until the velocity of the wheel reaches 70km/h. Once it reaches the velocity limit, the braking operation is started. There is another velocity threshold which states the minimum velocity level for applying ABS control algorithms. Under this minimum value of the velocity, the system becomes unstable if ABS algorithm is applied. Under such a circumstance, the maximum braking torque should be applied to the wheels without considering the target value of slip.

Fig. 7 and Fig. 8 present the system responses for constant and velocity dependent reference wheel slip values, respectively. The step size of GSMC is set to 20. Although the response of SMC is acceptable, the grey predictor is better under noisy conditions. This indicates that grey predictive controllers are more robust in real-time systems that are subjected to noise from both inside and outside of the system. There is no drastic change in stopping distance values between SMC and GSMC.

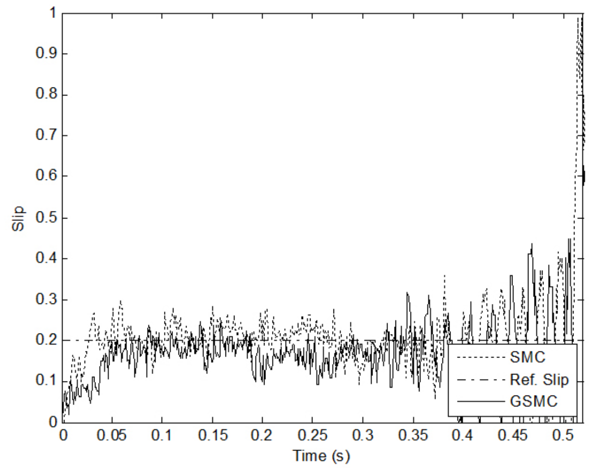

Fig. 7. Wheel slip of SMC and GSMC for constant reference

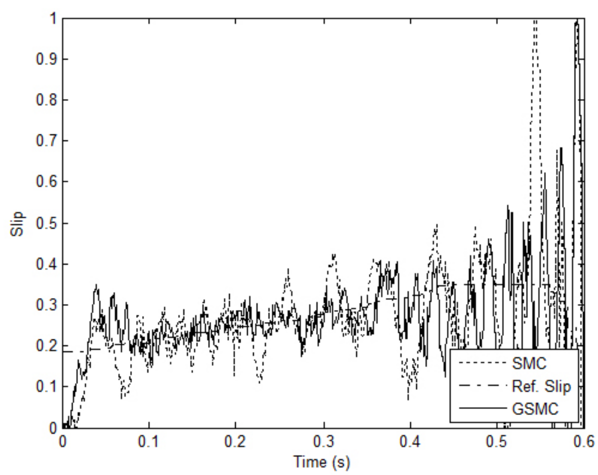

Fig. 8. Wheel slip of SMC and GSMC for velocity dependent reference

VI. CONCLUSION

In this study, a grey sliding mode control algorithm for ABS system has been proposed. According to various simulation and real time experimental results, the most attractive characteristic of the proposed controller is the robustness in the presence of the uncertainties in the system, such as noisy measurements or disturbances. The proposed grey controller has the ability to handle these difficulties. Hence, braking operation will be more stable and the performance of ABS system will be increased. Encouraged by these results, further experimental investigations for different road conditions are about to be launched.

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