A Grey System Modeling Approach for Sliding-Mode Control of Antilock Braking System

Erdal Kayacan, *Student Member, IEEE*, Yesim Oniz, *Student Member, IEEE*, and Okyay Kaynak, *Fellow, IEEE*

***Abstract*—In this paper, a grey sliding-mode controller is proposed to regulate the wheel slip, depending on the vehicle forward velocity. The proposed controller anticipates the upcoming values of wheel slip and takes the necessary action to keep the wheel slip at the desired value. The performance of the control algorithm as applied to a quarter vehicle is evaluated through simulations and experimental studies that include sudden changes in road conditions. It is observed that the proposed controller is capable of achieving faster convergence and better noise response than the conventional approaches. It is concluded that the use of grey system theory, which has certain prediction capabilities, can be a viable alternative approach when the conventional control methods cannot meet the desired performance specifications.**

***Index Terms*—Antilock braking system (ABS), GM(1,1), grey system theory, sliding-mode control.**

I. Introduction

ACTIVE SAFETY systems such as antilock braking system (ABS) and electronic stability programs significantly reduce the number of road accidents [1]. ABS is an electronically controlled system that helps the driver to maintain control of the vehicle during emergency braking while preventing the wheels to lock up. Furthermore, by keeping brake pressure just below the point of causing a wheel to lock, ABS ensures that the maximum braking power is used to stop the vehicle, and the minimum possible stopping distance is achieved [2]. In a conventional ABS, the angular velocity of the wheels and the linear acceleration of the vehicle are measured with sensors. The data are used to make the decision, whether the wheel is about to lock. If a tendency toward wheel block is perceived, the pressure in the brake cylinder is reduced. After the wheels are prevented from locking, the pressure in the cylinder is increased again [3].

During accelerating or braking, the generated friction forces are proportional to the normal load of the vehicle. The coefficient of this proportion is called road adhesion coefficient, and it is denoted by μ. Studies show that μ is a nonlinear function of wheel slip λ [4]. The typical μ–λ curve is obtained from the data of numerous experiments. Most of the ABS controllers are expected to keep the vehicle slip at a particular level, where the corresponding friction force (i.e., road adhesion coefficient) reaches its maximum value. Zanten states in [5] that the wheel slip should be kept between 0.08 and 0.3 to achieve optimal performance. Furthermore, some research papers show that the reference wheel slip does not have to be a constant value. In [6], the reference wheel slip is considered as a nonlinear function of some physical variables, including the velocity of the vehicle.

The accuracy of the control methods for ABS varies widely, depending on the system design, road conditions, and the driver's response. Most of these systems are based on empirical data and are heavily dependent on testing environment [7]. Although many attempts have been made over the decades, an accurate mathematical model of ABS has not been obtained yet. One of the shortcomings is that the controller must operate at an unstable equilibrium point for optimal performance. A small perturbation of the controller input may induce a drastic change in the output. Furthermore, at present, there are no affordable sensors which can accurately identify the road surface and make this data available to ABS controller. Regarding the fact that the system parameters highly depend on the road conditions and vary over a wide range, the performance of ABS may not always be satisfactory. Moreover, sensor signals are usually highly uncertain and noisy [8].

Because of the highly nonlinear and uncertain structure of ABS, many difficulties arise in the design of a wheel slip regulating controller. Sliding-mode control is a preferable option, as it guarantees the robustness of the system against changing working conditions. The stability requirements for switching surface are described in [9]. In [10] and [11], it is assumed that the optimal value of wheel slip, which will result in the maximum braking torque, is known. Drakunov *et al.* [12] employs sliding mode to achieve the maximum value of friction force without using *a priori* knowledge of optimum slip value. Kachroo and Tomizuka proposed a sliding-mode controller (SMC) in [13] that can maintain the wheel slip at any desired value. Unsal and Kachroo [14] proposed a sliding-mode observer to track the reference wheel slip, and a PI-like controller is used to reduce the chattering problem. Schinkel and Hunt employ in [15] a sliding-mode-like approach for ABS controller, in which two uncertain linear systems are used to deal with the inherently nonlinear dynamics of the vehicle. A sliding-mode observer is proposed in [16] to estimate the internal friction state of LuGre model, which is commonly used as a friction model in ABS. A self-learning fuzzy controller is combined

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with an SMC in [17]. The stability of the system is guaranteed, as the tuning algorithms for the controller are derived in the Lyapunov sense. A variable parameter SMC is developed for a four-wheel vehicle model, including the effects of tire slip, in [18]. Large pressure changes are compensated with the variable sliding parameter that is proportional to the square root of the derivative of brake pressure at the wheels. In [19], a robust fuzzy SMC for active suspensions of a nonlinear half-car model is proposed. A nonchattering sliding-mode control method is combined with a single-input–single-output fuzzy logic controller to improve its performance.

Aside from the sliding-mode control, there are several intelligent control schemes, including fuzzy logic control, adaptive control, and neural network approach. Mauer proposed a fuzzy controller in [20] to identify the condition of the road surface. The controller adjusts the braking torque based on the current and past values of wheel slip and brake pressure. In [21], the unknown road characteristics are resolved by a road estimator. In the controller design, an observer-based direct adaptive fuzzy neural controller is developed to tune the weighting factors of the neural controller online. Layne *et al.* [22] also adopted the fuzzy approach and modeled the plant to be controlled as a first-order linear dynamic system. Will *et al.* [23] developed a hybrid nonlinear control system that combines the sliding-mode-based observer with a proportional–integral derivative controller. The proposed controller does not require the information of surface type and can compute the optimal value of wheel slip online based on the data from longitudinal accelerometers and wheel speed sensors. Lee and Zak [24] have designed an ABS controller using genetic neural fuzzy control. While a neural optimizer is employed to find the optimal wheel slip, the fuzzy component computes the braking torque required to track the optimal slip. A genetic algorithm is used to adjust the parameters of the fuzzy logic component.

Grey system theory was first introduced in early 1980s by Professor Deng Ju-long. The theory has since then become quite popular with its ability to deal with the systems that have partially unknown parameters. It is therefore a good candidate to real-time control systems. For instance, in [25], a Li-ion battery is considered as a grey system, and a grey prediction technique is used to develop a grey-predicted Li-ion battery charge system. A further experimental example is presented in [26], which introduces an adaptive grey control system for a linear piezoelectric ceramic motor. Since the dynamic characteristics and motor parameters of the motor are highly nonlinear and time varying, an adaptive grey control system is therefore used to achieve high-precision position control under wide operation range. In this paper, a grey predictor is used to forecast the values the angular velocity of the wheels and the linear velocity of the vehicle for use in the SMC.

The motivation of this paper is to propose a grey SMC (GSMC) for tracking a reference wheel slip, which is a velocity-dependent variable. Due to highly nonlinear and uncertain characteristics of ABS, a grey predictor is employed to anticipate the future outputs of the system using the current data available. The grey predictor estimates the forthcoming value of wheel slip and the reference wheel slip, and SMC takes the necessary action to maintain wheel slip at the desired value. To observe the performance of the designed controller on various road conditions, a sudden change in the reference wheel slip is introduced. In the next section, a simplified quarter vehicle model is described. SMC and grey predictor are developed in Sections III and IV, respectively. Simulation and experimental results are provided and compared in Section V. Section VI makes some concluding remarks.

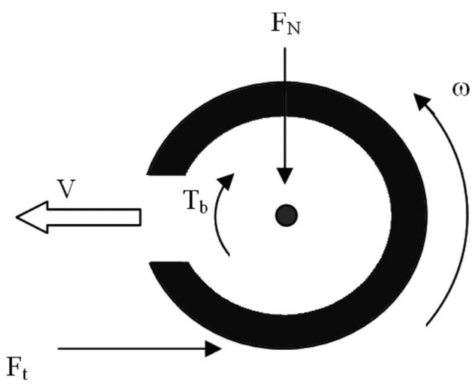

Fig. 1. Quarter vehicle model.

II. Mathematical Model

A. *Quarter Vehicle Model*

ABS can be considered as a mechatronic system which consists of some electromechanical components, sensors, actuators, regulation, etc. [27]. Fig. 1 shows the free body diagram of the quarter vehicle model describing the longitudinal motion of the vehicle and angular motion of the wheel under braking operation. Although the model is quite simple, it preserves the fundamental characteristics of an actual system [2]. In deriving the dynamic equations of the system, several assumptions are made. First, only longitudinal dynamics of the vehicle are considered. The lateral and vertical motions are neglected. Second, rolling resistance force is ignored, as it is very small due to braking. Furthermore, it is assumed that there is no interaction between the four wheels of the vehicle.

Table I shows the system parameters used in this paper.

B. *Vehicle Dynamics*

By the use of Newton's second law, the equation of the motion of the simplified vehicle can be expressed as

$$m\dot{V} = -F_t - F_a. \tag{1}$$

The aerodynamic force F_a depends on the shape, size, and instantaneous linear velocity of the vehicle [28], [29]

$$F_a = \frac{1}{4}\left(\frac{\rho}{2} C_d A_f V^2\right). \tag{2}$$

The numerical values used in (2) are $\rho = 1.23$ (kg/m^3), $C_d = 0.54$, and $A_f = 2.04$ (m^2).

The road friction force F_t can be given by the Coulomb law

$$F_t = \mu(\lambda, V) F_N. \tag{3}$$

TABLE I
NOMENCLATURE

Name	Description
m	Total mass of quarter vehicle
V	Vehicle velocity
F_t	Road friction force
F_a	Aerodynamic force acting on the vehicle
ρ	Mass density of the air
C_d	Vehicle drag coefficient
A_f	Vehicle frontal area
F_N	Total normal load
R	Wheel radius
μ	Road adhesion coefficient
J	Wheel moment of inertia
w	Wheel speed
T_t	Tire tractive torque
T_b	Braking torque
λ	Wheel slip
λ_R	Reference slip
s	Switching function
T_b, eq	Equivalent control brake torque
T_b, h	Striking control brake torque
η_s	Design parameter
σ_0	Rubber longitudinal stiffness
σ_1	Rubber longitudinal damping
σ_2	Viscous relative damping
μ_s	Normalized static friction coefficient
μ_c	Normalized Coulomb friction coefficient
V_s	Stribeck relative velocity
V_r	Relative velocity

C. Wheel Dynamics

During deceleration, a braking torque is applied to the wheels, which causes wheel and vehicle speeds to decrease. The rolling resistance force of the wheel is much smaller than the friction force between the wheel and road, and hence, it can be neglected. According to Newton's second law, the equation of the motion of the wheel can be written as

$$J\dot{w} = -T_b + F_t R. \tag{4}$$

Under normal operating conditions, the rotational velocity of the wheel would match the forward velocity of the car. When the brakes are applied, braking forces are generated at the interface between the wheel and the road surface, which causes the wheel speed to decrease. As the force at the wheel increases, slippage will occur between the tire and the road surface, and the wheel speed will tend to be lower than the vehicle speed. The parameter used to specify this difference in these velocities during braking is called wheel slip λ, and it is defined as

$$\lambda = \frac{V - wR}{V}. \tag{5}$$

While a wheel slip of zero indicates that the wheel and vehicle velocities are the same, a ratio of one implies that the tire is not rotating and the wheels are skidding on the road surface, i.e., the vehicle is no longer steerable.

The road adhesion coefficient is a nonlinear function of some physical variables, including the velocity of the vehicle and wheel slip. The LuGre friction model [6] deals with the

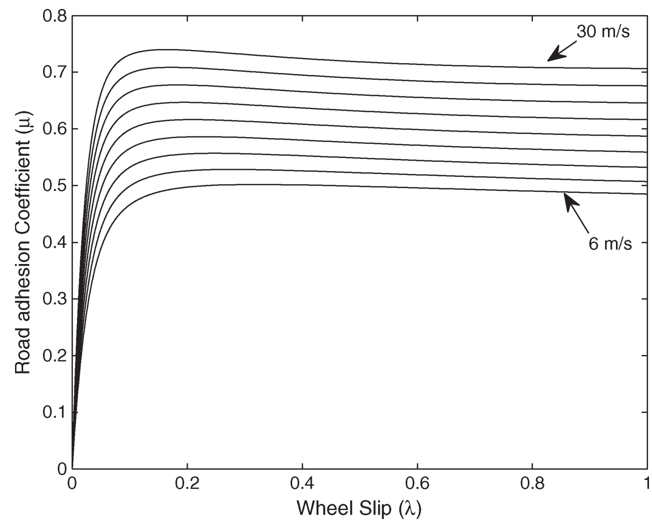

Fig. 2. μ–λ relation for several velocity values for dry road condition.

dependence of friction on velocity. In [30], a pseudostatic expression for friction force is given as

$$\mu(\eta, V) = -h\left[1 + 2\gamma\frac{h}{\sigma_0 L|\eta|}\left(e^{-\frac{\sigma_0 L|\eta|}{2h}} - 1\right)\right] - \sigma_2 V_r \tag{6}$$

where

$$\eta = \frac{\lambda}{\lambda - 1} \qquad h = \mu_c + (\mu_s - \mu_c)e^{-\left|\frac{V_r}{V_s}\right|^{1/2}}$$
$$\gamma = 1 - \frac{\sigma_1|\eta|}{Rwh} \qquad V_r = wR - V.$$

The numerical values used in (6) are

$$\begin{aligned} \sigma_0 &= 100\ (1/\mathrm{m}) \\ \sigma_1 &= 0.7\ (\mathrm{s/m}) \\ \sigma_2 &= 0.011\ (\mathrm{s/m}) \\ \mu_s &= 0.5 \\ \mu_c &= 0.35 \\ V_s &= 10\ (\mathrm{m/s}) \\ L &= 0.25\ (\mathrm{m}). \end{aligned}$$

As can be seen from (6), if the velocity of the vehicle changes, the friction coefficient μ will change with the wheel slip λ. Fig. 2 shows this relationship for different velocity values from 6 to 30 m/s for every 3 m/s increment for a dry road surface. The peak values of the curves are used as the reference wheel slip values in the proposed control system.

III. SMC

Sliding-mode control is a simple form of robust control which provides an effective method to control nonlinear plants. The design procedure of sliding-mode control methodology consists of two main steps: First, a sliding surface that models the desired closed-loop performance is chosen, and then, the control law, such that the system state trajectories are forced toward the sliding surface, is derived [31]. Once the sliding surface is reached, the system state trajectories should stay on it.

In this paper, SMC is used to track the reference wheel slip. Hence, the switching function s is defined as [2]

$$s = \lambda - \lambda_R \tag{7}$$

where λ_R is the reference input wheel slip. The sliding motion occurs when the state $(\lambda_R, \dot{\lambda}_R)$ reaches the switching subspace (a point in this case) defined by $s = 0$. The control that keeps the state on the switching subspace is called the equivalent control. In this paper, it is called equivalent control brake torque $T_{b,\mathrm{eq}}$. The dynamics in sliding mode can be written as

$$\dot{s} = 0. \tag{8}$$

It can be shown that, in the derivation of (9), (1)–(4) are used

$$\dot{\lambda} = \frac{1}{V}\left[\frac{-R}{J}(F_t R - T_b) + (1-\lambda)\dot{V}\right]. \tag{9}$$

If it is assumed that the reference wheel slip is constant, (7) and (8) result

$$\dot{\lambda} = 0. \tag{10}$$

Substituting (9) into (10) gives

$$0 = \frac{1}{V}\left[\frac{-R}{J}(F_t R - T_b) + (1-\lambda)\dot{V}\right]. \tag{11}$$

Solving for the equivalent control brake torque $T_{b,\mathrm{eq}}$ gives

$$T_{b,\mathrm{eq}} = F_t R - (1-\lambda)\frac{\dot{V}J}{R}. \tag{12}$$

If the system state $(\lambda, \dot{\lambda})$ is not on the switching subspace, an additional control term called hitting control brake torque $T_{b,h}$ should be added to the overall brake torque control signal. When the system state is on the switching subspace, the hitting control is zero. The hitting control brake torque $T_{b,h}$ is determined by the following reaching condition, where η_s is a strictly positive design parameter:

$$s\dot{s} \le -\eta_s |s|. \tag{13}$$

Using (7) and (9), (13) can be rewritten as

$$s\dot{\lambda} \le -\eta_s |s|. \tag{14}$$

Substitution of (9) into (14) results

$$\frac{s}{V}\left(\frac{-R}{J}\left(F_t R - (T_{b,\mathrm{eq}} - T_{b,h}\mathrm{sgn}(s))\right) + (1-\lambda)\dot{V}\right) \le -\eta_s |s|. \tag{15}$$

Solving for the hitting control brake torque $T_{b,h}$ results in (16), where $F \ge ((1-\lambda)|\dot{V} - \hat{\dot{V}}|)/V$ and $\hat{\dot{V}}$ is the estimated value of vehicle acceleration

$$T_{b,h} = \frac{VJ}{R}(F + \eta_s). \tag{16}$$

The overall brake torque control is assumed to have the form

$$T_b = T_{b,\mathrm{eq}} - T_{b,h}\mathrm{sgn}(s). \tag{17}$$

Because the control effort is discontinuous and creates chattering in SMC, it may excite the high-frequency dynamics [32]. To deal with the chattering problem, the discontinuous switching function is replaced by the following continuous one, where $\delta > 0$ [33]:

$$f(s) = \frac{s}{|s| + \delta}. \tag{18}$$

Thus, the brake torque control can be expressed as

$$T_b = F_t R - (1-\lambda)\frac{\hat{\dot{V}}J}{R} - \frac{VJ}{R}(F + \eta_s) f(s). \tag{19}$$

IV. GSMC

A. Basics of Grey System Theory

In control theory, a system can be defined with a color that represents the amount of clear information about that system. For instance, a system can be called as "black box" if its internal characteristics or mathematical equations that describe its dynamics are completely unknown. On the other hand, if the description of the system is completely known, it can be named as white system. Similarly, a system that has both known and unknown information is defined as a grey system. In real life, every system can be considered as a grey system because there are always some uncertainties [34].

B. Differences Among Probability and Statistics, Fuzzy Theory, and Grey Theory

Although probability and statistics, fuzzy theory, and grey system theory deal with uncertain information, different methods and mathematical tools are used to analyze the data. While fuzzy mathematics mainly deals with problems associated with cognitive uncertainty by experience with the help of affiliation functions, probability and statistics need special distributions and samples of reasonable size to draw inferences. These very different approaches have a serious difficulty in such situations either without any prior experience or without satisfying any special distributions and with small sample size [35]. Grey system theory and grey controllers have great advantages in such kinds of systems, because grey controllers have the ability to handle the uncertain information and use the data effectively. Grey controllers investigate the behavioral characteristics of a system using a sequence of definite white numbers. The characteristic data obtained from the system are supposed to contain, if there are, the laws of development of the system.

C. GM(1,1) Model

The GM(1,1) type of grey model is most widely used in the literature, which stands for "Grey Model First-Order One Variable." This model is a time series forecasting model. The differential equations of the GM(1,1) model have time-varying coefficients. In other words, the model is renewed as the new data become available to the prediction model.

In order to smooth the randomness, the primitive data obtained from the system to form the GM(1,1) are subjected to an operator, named accumulating generation operator (AGO) [36]. The differential equation [i.e., GM(1,1)] thus evolved is solved to obtain the n-step ahead the predicted value of the system. Finally, using the predicted value, the inverse AGO (IAGO) is applied to find the predicted values of the primitive data.

Consider a single-input–single-output system. Assume that the time sequence $X^{(0)}$ represents the outputs of the system

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\right), \qquad n \geq 4 \tag{20}$$

where $X^{(0)}$ is a nonnegative sequence, and n is the sample size of the data. When this sequence is subjected to AGO, the following sequence $X^{(1)}$ is obtained. It is obvious that $X^{(1)}$ is monotonically increasing

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right), \qquad n \geq 4 \tag{21}$$

where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \qquad k = 1, 2, 3, \ldots, n. \tag{22}$$

The generated mean sequence $Z^{(1)}$ of $X^{(1)}$ is defined as

$$Z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), \ldots, z^{(1)}(n)\right) \tag{23}$$

where $z^{(1)}(k)$ is the mean value of adjacent data, i.e.,

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), \qquad k = 2, 3, \ldots, n. \tag{24}$$

The least square estimate sequence of the grey difference equation of GM(1,1) is defined as follows [36]:

$$x^{(0)}(k) + az^{(1)}(k) = b. \tag{25}$$

The whitening equation is therefore as follows:

$$\frac{dx^{1}(t)}{dt} + ax^{(1)}(t) = b. \tag{26}$$

In (26), $[a, b]^{\mathrm{T}}$ is a sequence of parameters that can be found as follows:

$$[a, b]^{\mathrm{T}} = (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}Y \tag{27}$$

where

$$Y = \left[x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\right]^{\mathrm{T}} \tag{28}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ -z^{(1)}(n) & 1 \end{bmatrix}. \tag{29}$$

According to (25), the solution of $x^{(1)}(t)$ at time k is

$$x_p^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a}. \tag{30}$$

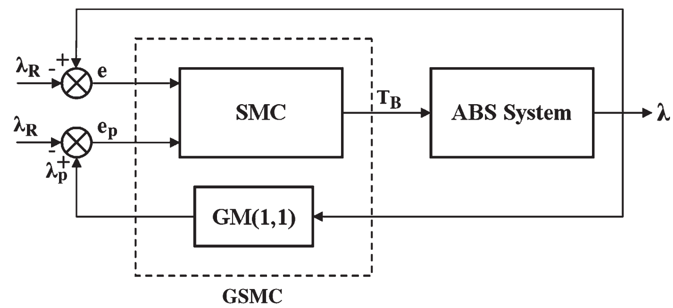

Fig. 3. Structure of GSMC.

To obtain the predicted value of the primitive data at time $(k+1)$, IAGO is used to establish the following grey model:

$$x_p^{(0)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak}(1 - e^{a}) \tag{31}$$

and the predicted value of the primitive data at time $(k+H)$ is

$$x_p^{(0)}(k+H) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-a(k+H-1)}(1 - e^{a}). \tag{32}$$

D. Design of GSMC

To integrate grey prediction into the proposed SMC, a new time-varying grey switching subspace $s(\lambda, t)$ is defined as

$$s(\lambda, t) = e + e_p \tag{33}$$

where $e_p = \lambda_p - \lambda_R$ is a value predicted by GM(1,1) model, $e = \lambda - \lambda_R$, λ_p is the predicted value of wheel slip, and λ_R is the reference wheel slip. This idea has been investigated in the literature earlier [37], the design of the GSMC controller being based on a sliding surface which is a line. In this paper, a simpler approach is proposed, defining the time-varying grey switching subspace $s(\lambda; t)$ as a point as indicated earlier. If the Lyapunov candidate function V_L is defined as

$$V_L = \frac{1}{2}\left(s(\lambda, t)\right)^2 \tag{34}$$

then, it is guaranteed that the tracking error of GSMC will be less than the one of conventional SMC [37]. The structure of the proposed GSMC is shown in Fig. 3. Some approaches seen in the literature [38] use a grey predictor to forecast the upcoming values of the error, whereas in this paper, the wheel slip values are predicted.

V. Simulation and Experimental Results

A. Simulation Results

To investigate the performance of the proposed control algorithms, a number of computer simulated dynamic responses are obtained. As mentioned in Section II, a pseudostatic curve is used to calculate the reference wheel slip and corresponding tire friction coefficient. These values are used to construct a table, which relates the vehicle speed to the peak values of the tire road friction coefficient and to the reference wheel slip.

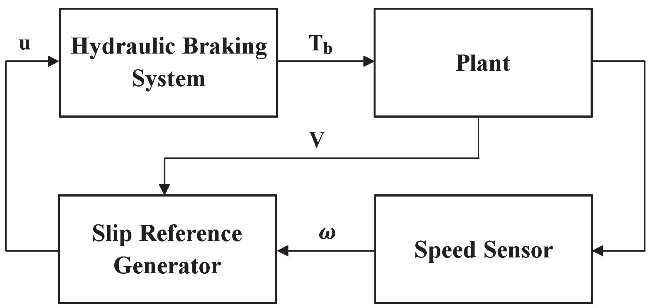

Fig. 4. Simulation scheme of ABS control.

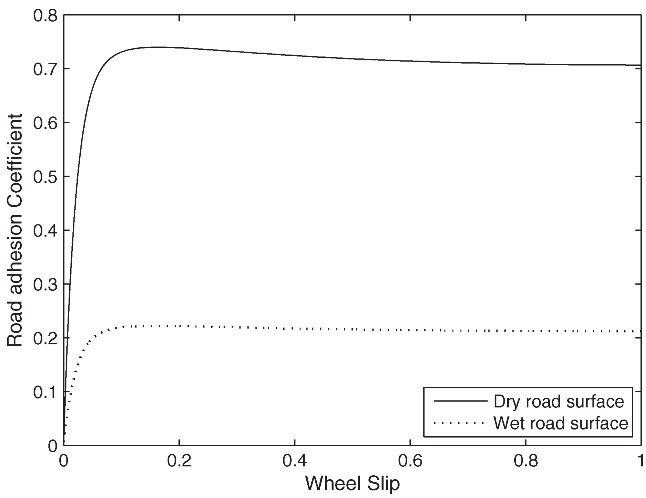

Fig. 5. μ–λ relation for dry and wet road conditions $(V = 30\text{ m/s})$.

Next, the reference wheel slip is made available to the proposed controllers at each step of the control loop. Fig. 4 shows the basic simulation structure of ABS control system in this paper.

All figures that follow show the simulation results for a car with initial longitudinal velocity of $V = 30$ m/s, maneuvering on a straight line. To simulate sudden changes on the road conditions, two different μ–λ curves are used, as shown in Fig. 5. First, the simulation starts on a dry road, then the vehicle suddenly enters in a wet road.

The step size of the grey predictor H has a deep impact on the effectiveness of the grey controllers. A grey predictor with a small fixed forecasting step size will make the system respond faster but cause larger overshoots. Conversely, the bigger step size of the grey predictor will cause overcompensation, resulting in a slow system response. The step size of the grey predictor should therefore be tuned by a trial and error operation or an adaptive step size algorithm should be used. In this paper, the step size is selected by trial and error.

To imitate a sudden change in the road conditions, the reference wheel slip corresponding to dry road is changed to the one of slippery road at $t = 2.5$ s. Figs. 6 and 7 show the system responses for SMC and GSMC. It can be seen that both controllers can track the reference input satisfactorily. The mechanical time delay is assumed to be 100 ms (a higher value of the mechanical time delay causes the system to have a bigger overshoot), and T_s is selected as 10 ms. In both controllers, the design parameter η_s has a value of 0.8, and δ is 0.07. From the responses obtained, it can be concluded that

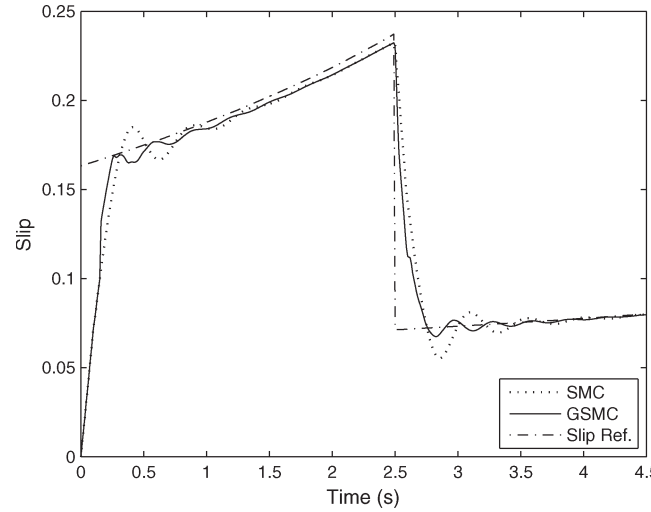

Fig. 6. Wheel slip for SMC and GSMC; $H = 8$.

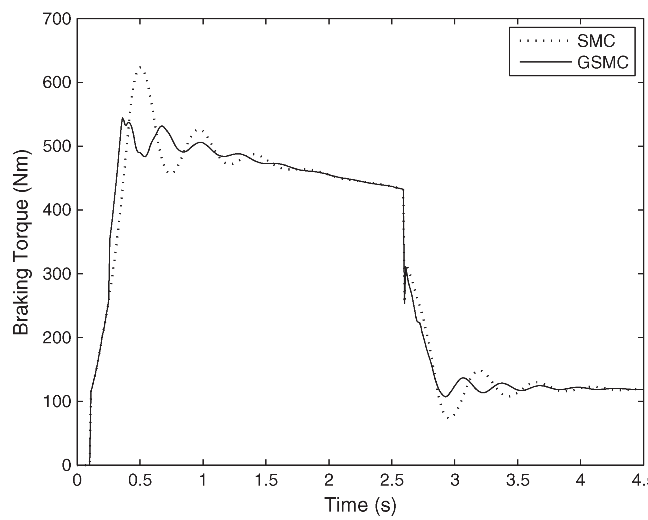

Fig. 7. Braking torque for SMC and GSMC; $H = 8$.

GSMC possesses shorter settling time and smaller overshoot. Moreover, GSMC has less oscillatory braking torque response than the conventional SMC. Fig. 8 shows the braking distance for both controllers, which are very close to each other.

In Fig. 9, band-limited white noise is added to the system at slip measurement. The noise power, which is the height of the power spectral density of the white noise, is equal to 5×10^{-7}. Although the response of SMC is acceptable, the grey predictor is better under noisy conditions. This indicates that grey predictive controllers are possibly more robust in real-time applications that are subjected to noise from both inside and outside of the system. Fig. 10 shows the zoomed view of the noise responses of the ABS model to SMC and GSMC between 1.5–2.5 s. It can be inferred that GSMC exhibits smaller variations from the desired slip value when compared to the conventional SMC.

A similar characteristic to Fig. 9 may be observed in Fig. 11. Grey predictor coupled to a conventional SMC controller exhibits fewer fluctuations compared to the conventional SMC. Hence, braking operation will be more stable, and the performance of the ABS will be increased. Fig. 12 shows the zoomed view of Fig. 11.

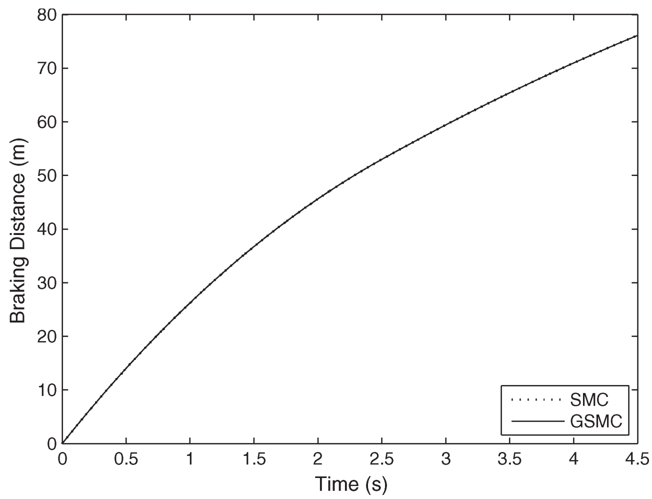

Fig. 8. Braking distance for SMC and GSMC; $H = 8$.

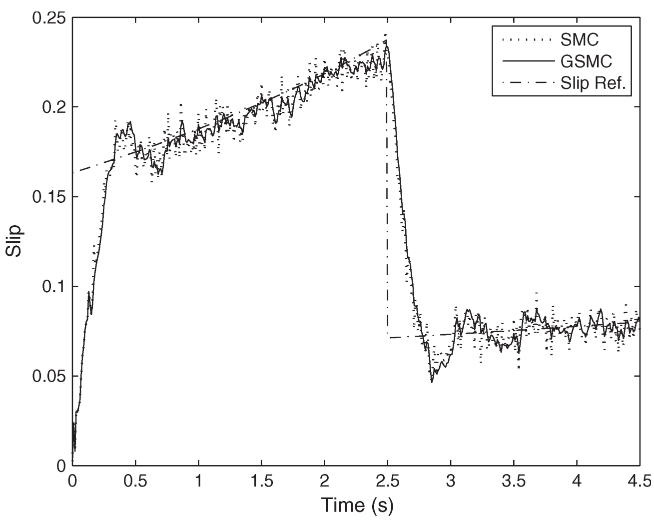

Fig. 9. Wheel slip for SMC and GSMC with white noise at the output measurement.

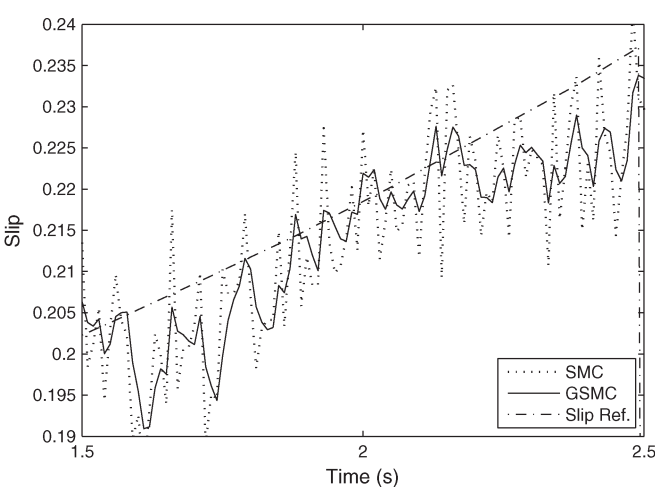

Fig. 10. Zoomed view of Fig. 9.

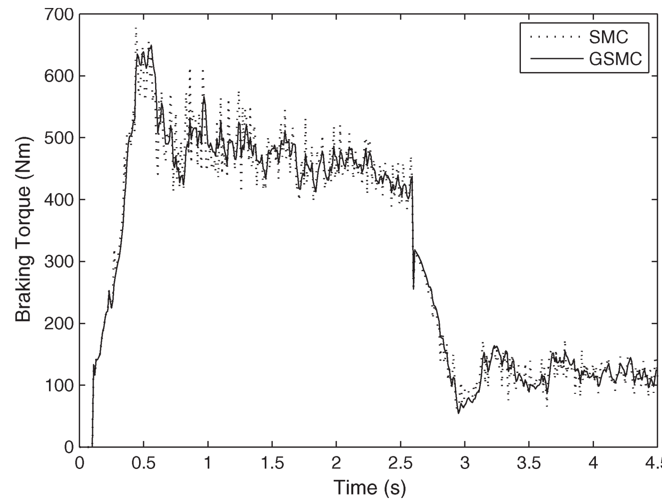

Fig. 11. Braking torque for SMC and GSMC with white noise at the output measurement.

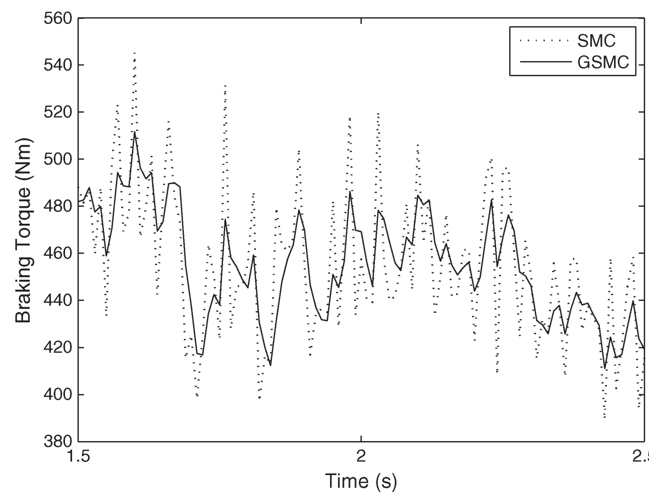

Fig. 12. Zoomed view of Fig. 11.

B. Experimental Results

To validate the results obtained in the simulations, a simple real-time application is conducted on a laboratory experimental setup [39]. The setup consists of two rolling wheels. The lower wheel, made of aluminium, imitates relative road motion, whereas the upper wheel, mounted to the balance lever, animates the wheel of the vehicle. In order to accelerate the lower wheel, a large flat dc motor is coupled on it. The upper wheel is equipped with a disk brake system that is driven by a small dc motor. There are three identical encoders measuring the rotational angles of two wheels and the deviation angle of the balance lever of the car wheel. To imitate the behavior of the vehicle during braking on a dry and straight road, the wheel is accelerated until the velocity of the wheel reaches 70 km/h, and then, the braking operation is started. It should be noted that, on the available setup, the experiments can be run for a specific road condition (just for dry road), and therefore, no experimental results could have been carried out for changing road conditions. On the other hand, to be able to compare the experimental results with the simulation results in the previous section, the reference wheel slip has been changed suddenly to a lower value. Hence, the change in road conditions, i.e., from dry asphalt to icy road, has been imitated.

Fig. 13 shows the system response for velocity-dependent reference wheel slip. The experimental results have indicated

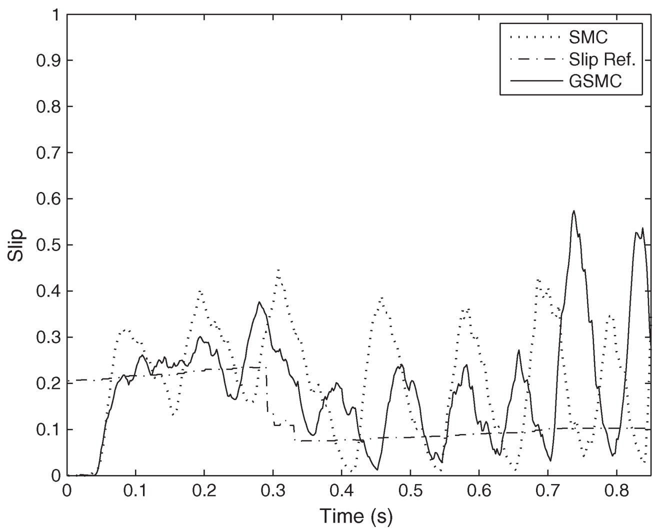

Fig. 13. Experimental result for a velocity-dependent reference.

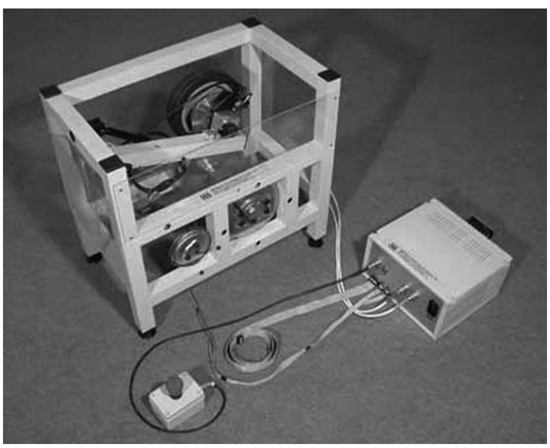

Fig. 14. Inteco ABS setup.

that the main differences in the performances of SMC and GSMC are in the noise response and the overshoot. The results obtained are consistent with the simulation studies. The equilibrium point of ABS is an unstable equilibrium point, and when the velocity of the car is below a threshold value, it is very difficult to force the slip to remain at a constant value. In real life cases, there is a relay-type switch for the braking system of the vehicle. If the velocity of the vehicle decreases to a minimum value, the ABS controller is switched off (Fig. 14), and regular brakes are applied. Once the system trajectory is close to the unstable equilibrium point, the oscillations are magnified because of the reasons mentioned earlier.

To demonstrate the accuracy of the proposed controllers, the following accuracy evaluation standard is used:

$$TE = \sum_{i=1}^{N} e[i]^2 \tag{35}$$

where TE, $e[i]$, and N are the total error, the system error at each time step i of the control algorithm, and the number of samples, respectively. In the real-time experiment given in Fig. 10, GSMC results in 18.53% better noise response when compared to conventional SMC wrt TE defined in (35).

VI. CONCLUSION

In this paper, an SMC and an SMC with a grey predictor for ABS system have been proposed, and their performances have been compared both by simulation and experimental studies. The simulation results obtained indicate that the most attractive characteristic of the latter controller is the robustness against the uncertainties in the system, such as noisy measurements or disturbances. The proposed grey controller has the ability to handle these difficulties. Hence, the braking operation is more stable, resulting in an increase in the performance of ABS. It is also observed that GSMC possesses shorter settling time and smaller overshoot in both the slip and the braking torque. The improvements observed in performance during the simulation studies are validated through real-time experiments on a laboratory setup, albeit for a specific road condition. The experimental results show that the GSMC can effectively suppress oscillations (caused by both the internal and external noises that the system is subjected to) both in the slip response and the braking torque. Encouraged by these simulation and experimental results, further investigation on the performance of the proposed controllers for different road conditions and more complex ABS models is about to be launched, including the adjustment of the step size H of the predictor adaptively.

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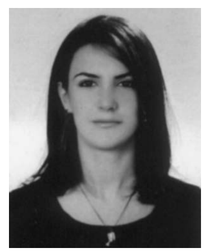

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