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Product-sum-type fuzzy controllers are known to have similar characteristics to PD-type controllers. In the case of type-0 control systems, PID-type fuzzy controllers have been proposed in the literature in order to eliminate the steady-state error. However, these control methods, essentially based on conventional PID theory, have no predictive capabilities. The concept of grey system theory, which has a certain prediction capability, offers an alternative approach for various kinds of conventional control methods, such as PID control and fuzzy control. This paper proposes a grey prediction-based fuzzy PID controller that can overcome the stated shortcomings. In order to obtain a better controller performance, another fuzzy controller is designed to change the step size of the grey predictor. A non-linear liquid level system is taken as a test bed. The grey model developed is examined under several different conditions and it is shown that the proposed grey fuzzy PID controller can predict the future output value of the system. It is clear that the proposed adaptive PID-type fuzzy controller is effective in controlling such a non-linear system accurately by changing the step size adaptively for real-time working.

Key words: adaptive step size; GM(1,1) model; grey fuzzy PID-type control; grey predictive control; grey predictors; liquid level system

1. Introduction

In control theory, a system can be defined with a colour that represents the amount of clear information about that system. For instance, a system can be called ''black box'' if the internal characteristics or mathematical equations that describe its dynamics are

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completely unknown. On the other hand, if the description of the system is completely known, it can be called a ''white system''. Similarly, a system that has both known and unknown information is defined as a ''grey system''. In real life, because of noise from both inside and outside of the system (and the limitations of our cognitive abilities!), the information we can learn about that system is always uncertain and limited in scope (Lin and Liu, 2004). Therefore, every system can be considered a grey system in real life because there are always some uncertainties.

There are many situations in industrial control systems when control engineers face the difficulty of incomplete or insufficient information. The reason for this is because of the lack of modelling information or the fact that the right observation and control variables have not been employed. For instance, the data collected from a motor control system always contains some grey characteristics related to the time-varying parameters of the system and measurement difficulties. Similarly, it is difficult to forecast the electricity consumption of a region accurately because of various kinds of social and economic factors. These factors are generally random and make it difficult to obtain an accurate model.

Grey system theory was first introduced by Professor Deng Ju-long from China in the international journal *Systems and Control Letters* in 1982 (Deng, 1989). The theory is distinguished by its ability to deal with the systems that have partially unknown parameters. It is therefore a good candidate for use in real-time control systems. With the use of grey system mathematics (eg, grey equations and grey matrixes, etc.), it is possible to generate meaningful information using insufficient and poor data. Grey predictors have the ability to predict the future outputs of a system by using recently obtained data.

During the last two decades, grey system theory has developed rapidly and caught the attention of many researchers. It has been widely and successfully applied to various systems such as social, economic, financial, scientific and technological, agricultural, industrial, transportation, mechanical, meteorological, hydrological, medical, military, etc., systems. Some early milestones in control engineering area are as follows: a grey prediction controller, combined with a conventional controller for an unknown system was proposed by Cheng in 1986 (Cheng, 1986). In 1994, Huang proposed the basic structure of a grey prediction fuzzy model to control robotic motion and an inverted pendulum (which is a classical control problem; Huang and Hu, 1995; Huang and Huang, 1994). In these studies and the others, it has been seen that grey system theory-based approaches can achieve good performance characteristics when applied to real-time systems, since grey predictors adapt their parameters to new conditions as new outputs become available. Because of this, grey controllers are more robust with respect to noise, lack of modelling information and to other disturbances when compared with conventional controllers.

Although probability and statistics, fuzzy theory and grey system theory all deal with uncertain information, different methods and mathematical tools are used to analyse the data. While fuzzy mathematics mainly deals with problems associated with cognitive uncertainty with the help of affiliation functions, probability and statistics need special distribution functions and samples of reasonable size to draw inferences. With these two approaches, serious difficulties are therefore faced in handling situations when no prior experience is available or when the sample size is small (Liu and Lin, 1998). Under such conditions, grey system theory and grey controllers can provide some advantages, because they have the ability to handle uncertain information and use the data effectively. Grey controllers investigate the behavioural characteristics of a system using a sequence of definite white numbers. The characteristic data obtained from the system is expected to contain the physical laws of the system. The methods of probability and statistics study the uncertain data from a stochastic point of view. They focus on the statistical laws existing in the history of the uncertain data and the probability of data within possible outcomes (Lin and Liu, 2004).

The traditional grey predictor structure uses a fixed step size (Feng and Wong, 2002). A grey predictor with a small fixed forecasting step size will make the system respond faster but cause larger overshoots. Conversely, a bigger step size of grey predictor will cause overcompensation, resulting in a slow system response. In order to obtain a fast system response with little overshoot, the step size of the grey predictor should be changed adaptively. In the literature on grey system theory, there are some methods that tune the step size of the grey predictor according to the input state of the system (Wong and Liang, 1997). In order to determine the appropriate forecasting step size, some online rule-tuning algorithms using a fuzzy inference system have been proposed for the control of an inverted pendulum, a fuzzy tracking method for a mobile robot and non-minimum phase systems (Feng and Wong, 2002; Wong and Liang, 1997; Wong *et al*., 2001). In another paper, a Sugeno-type fuzzy inference system has been proposed for large time-delay systems (Han *et al*., 2005). In this work, a similar but simpler approach is proposed.

2. Fundamental concepts of grey system theory and GM(1,1) model

2.1 Grey system modelling

Grey numbers, grey algebraic and differential equations, grey matrices and their operations are used to deal with grey systems. A grey number is a number whose value is not known exactly but it takes values within a certain range. Grey numbers might have only upper limits, only lower limits or both. Grey algebraic and differential equations, and grey matrices all have grey coefficients.

2.2 Preliminaries

The GM(1, 1) model can only be used in positive data sequences (Deng, 1989). In this paper, a non-linear liquid level tank is considered. It is obvious that the liquid level in a tank is always positive, so the GM(1,1) model can be used to forecast the liquid level in this tank. A critical constraint of the grey model GM(1, 1) is that the ratio $x^{(0)}(k-1)/x^{(0)}(k)$ must be in the interval of [0.1345; 7.389] (Deng, 1989).

2.3 GM(n, m) model

Grey models can predict the future outputs of systems with high accuracy without a mathematical model of the actual system. In grey systems theory, GM(n,m) denotes a grey model, where n is the order of the difference equation and m is the number of variables. Although various types of grey models can be mentioned, most of the previous researchers have focused their attention on the GM(1, 1) model in their predictions because of its computational efficiency. It should be noted that in real-time applications, the computational burden is the most important parameter after the performance.

2.4 GM(1, 1) Model

The GM(1, 1)-type of grey model is most widely used in the literature, pronounced as the "grey model first order one variable". This model is a time-series forecasting model. The differential equations of the GM(1, 1) model have time-varying coefficients. In other words, the model is renewed as the new data become available to the prediction model.

In grey systems theory, the process of extracting the governing laws of a system using the available data is known as the generation of the grey sequence (Liu and Lin, 1998). It is argued that even though the available data of the system, which are generally white numbers, are too complex or chaotic, they always obey some governing laws. If the data obtained from a grey system is somehow smoothed, it is easier to derive any special characteristics of that system. In order to smooth the randomness, the primitive data obtained from the system to form the GM(1, 1) is subjected to an operator, named the accumulating generation operation (AGO; Deng, 1989), described below. The differential equation [ie, GM(1, 1)] thus evolved is solved to obtain the n-step ahead predicted value of the system. Finally, using the predicted value, the inverse accumulating operation (IAGO) is applied to find the predicted values of original data.

Consider a single-input and single-output system. Assume that the time sequence $X^{(0)}$ represents the outputs of the system:

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\right), \quad n \geq 4 \tag{1}$$

where $X^{(0)}$ is a non-negative sequence and n is the sample size of the data. When this sequence is subjected to the AGO, the following sequence $X^{(1)}$ is obtained. It is obvious that $X^{(1)}$ is monotone increasing.

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right), \quad n \geq 4 \tag{2}$$

where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, \ldots, n \tag{3}$$

The generated mean sequence $Z^{(1)}$ of $X^{(1)}$ is defined as:

$$Z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), \ldots, z^{(1)}(n)\right) \tag{4}$$

where $z^{(1)}(k)$ is the mean value of adjacent data, ie,

$$z^{(1)}(k) = \frac{1}{2}x^{(1)}(k) + \frac{1}{2}x^{(1)}(k-1), \quad k = 2, 3, \ldots, n \tag{5}$$

The least square estimate sequence of the grey difference equation of GM(1,1) is defined as follows (Deng, 1989):

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{6}$$

The whitening equation is therefore as follows:

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b \tag{7}$$

In above, $[a, b]^T$ is a sequence of parameters that can be found as follows:

$$\begin{bmatrix} a & b \end{bmatrix}^T = (B^T B)^{-1} B^T Y \tag{8}$$

where

$$Y = \left[x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\right]^T \tag{9}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ . & . \\ . & . \\ . & . \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{10}$$

According to Equation (7), the solution of $x^{(1)}(t)$ at time k:

$$x_p^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right]e^{-ak} + \frac{b}{a} \tag{11}$$

To obtain the predicted value of the primitive data at time $(k+H)$, the IAGO is used to establish the following grey model.

$$x_p^{(0)}(k+H) = \left[x^{(0)}(1) - \frac{b}{a}\right]e^{-a(k+H-1)}(1-e^{a}) \tag{12}$$

2.5 GM(1,1) rolling model

The GM(1, 1) rolling model is based on the forward data of a sequence to build the GM(1, 1). For instance, using $x^{(0)}(k)$, $x^{(0)}(k+1)$, $x^{(0)}(k+2)$ and $x^{(0)}(k+3)$, the model predicts the value of the next point $x^{(0)}(k+4)$. In the next few steps, the first point is always shifted to the second. This procedure is repeated until the end of the sequence; this method is called ''rolling check'' (Wen, 2004). A GM(1, 1) rolling model is used to predict the long continuous data sequences such as the outputs of a system, price of a specific product, trend analysis for finance statements or social parameters, etc. In this paper, a GM(1, 1) rolling model is used to predict the future outputs of the non-linear liquid level system.

3. Combining fuzzy and PID-type control

3.1 Analysis of the fuzzy controller

Consider a product-sum-type fuzzy controller with two inputs and one crisp output (MISO). Let the inputs to the fuzzy controller be the error e and the rate of change of the error $\acute{e}$, and the output of the fuzzy controller (ie, the input to the controlled process) be u. If an analysis of this controller is made, it can be seen that it behaves approximately like a PD controller. We can therefore consider it a time-varying parameter PD controller (Qiau and Muzimoto, 1996). Such a controller is named a PD-type fuzzy controller (PDFC) in the literature.

It is well known that if the controlled system is ''type 0'', a P- or PD-type controller cannot eliminate the steady-state error. Although the use of an integral term in the controller (such a PI controller) can take care of the steady-state error, it can deteriorate the transient characteristics by slowing the response. However, with a PID-type fuzzy controller, fast rise times and small overshoots as well as short settling times can be achieved with no steady-state error.

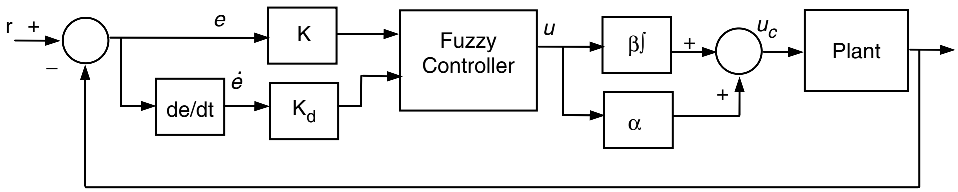

Figure 1 Block diagram of the PID-type fuzzy control system

3.2 PID-type fuzzy control

In order to design a PID-type fuzzy controller (PIDFC), one can design a fuzzy controller with three inputs: error, the rate of change of error and the integration of the error. Handling the three variables is, however, in practice, quite difficult. Besides, adding another input to the controller will increase the number of rules exponentially. This requires more computational effort, leading to a larger execution time.

Because of the drawbacks mentioned above, a PID-type fuzzy controller consisting of only the error and the rate of change of error is used in the proposed method. This allows PD- and PI-type fuzzy controllers to work in parallel (Qiau and Muzimoto, 1996; Woo *et al*., 2000).

An equivalent structure is shown in Figure 1, where β and α are the weights of the PI- and PD-type controllers, respectively. Similarly, K and K_d are the scaling factors for e and $\acute{e}$, respectively.

The output of the controller can be expressed as:

$$u_c = \alpha u + \beta \int u dt \tag{13}$$

As the α/β ratio becomes larger, the effect of the derivative control increases with respect to integral control (Engin *et al*., 2004). This controller is called as PID-type fuzzy controller (PIDFC).

4. Grey PID-type fuzzy control

4.1 Rule base and membership functions

In a conventional fuzzy inference system, an expert, who is familiar with the system to be modelled, decides on the number of rules. The fuzzy PID-type control rule base employed in this paper is shown in Table 1. The membership functions of error, rate of change of error and control signal, shown in Figure 2, are chosen as triangular membership functions.

Table 1 A general fuzzy PID-type rule base

	e						
é	NL	NM	NS	ZR	PS	PM	PL
PL	ZR	PS	PM	PL	PL	PL	PL
PM	NS	ZR	PS	PM	PL	PL	PL
PS	NM	NS	ZR	PS	PM	PL	PL
ZR	NL	NM	NS	ZR	PS	PM	PL
NS	NL	NL	NM	NS	ZR	PS	PM
NM	NL	NL	NL	NM	NS	ZR	PS
NL	NL	NL	NL	NL	NM	NS	ZR

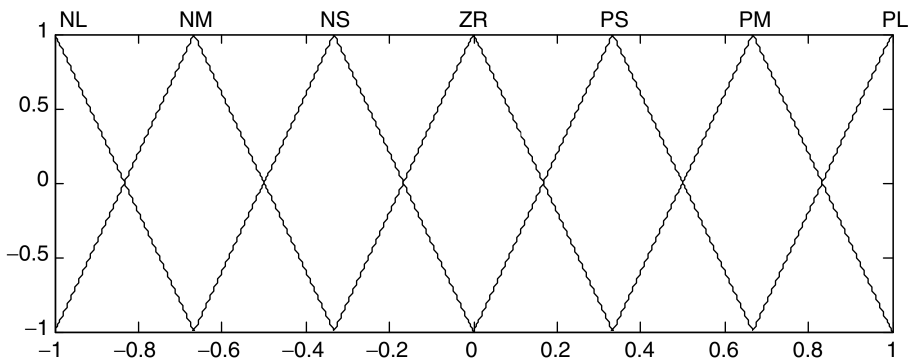

Figure 2 The membership functions of *e*, *é* and u

4.2 Design of grey PID-type fuzzy controller

In most control applications, the control signal is a function of the error present in the system at a prior time. This methodology is called ''delay control''. In grey systems theory, predicted error is used instead of current measured error (Wang *et al.*, 2003). Along similar lines, during the development of the grey PID-type fuzzy controller, the predicted error is considered the error of the system. The block diagram of the grey fuzzy PID control system with a fixed step size and the adaptive grey PID-type fuzzy controller with a variable step size proposed in this paper are showed in Figures 3 and 4, respectively.

In order to adapt the forecasting step size *H* to different states of the controller changing with error and the derivative of the error, another fuzzy controller is designed. The inputs of this fuzzy controller are *e* and *é*. The output variable is forecasting step size *H*. Triangle membership functions are used for the fuzzification process.

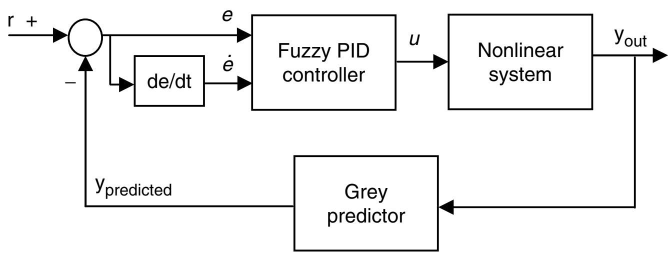

Figure 3 Block diagram of the grey PID-type fuzzy control system with a fixed step size

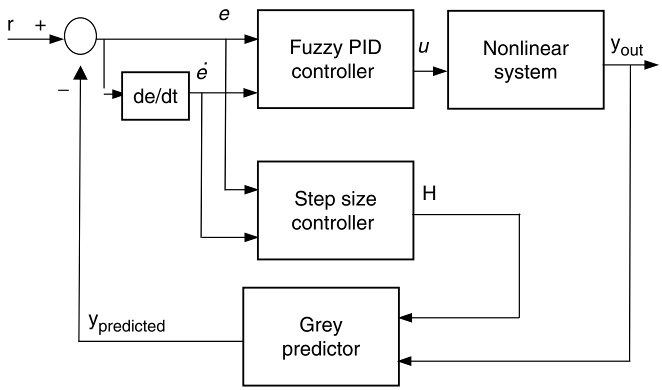

Figure 4 Block diagram of the adaptive grey fuzzy PID control system with a variable step size

Table 2 A general rule base for fuzzy step size controller

		e				
H		PL	PS	ZR	NS	NL
é	PL	VB	VB	B	MD	VS
	PS	VB	B	MD	S	S
	ZR	S	S	S	S	MD
	NS	S	S	MD	MD	VB
	NL	VS	MD	B	B	VB

The fuzzy variables *e* and *é* are partitioned into five subsets (NL, NS, ZR, PS and PL) and the output variable *H* is partitioned into five subsets (VS, S, MD, B, VB). The ranges of *e*, *é* and *H* are considered to be [–0.4; 1], [–0.05; 0.05] and [0; 60], respectively (Table 2).

5. Description of controlled object

A model of a non-linear liquid-level system will be obtained in this part of the paper (Doebelin, 1998). Figure 5 shows a simple system, the objective of which is to control the level of the liquid in a tank by adjusting the input flow rate in an effective way.

In this system, Q_{in} and Q_{out} are the maximum liquid flow rates in m^3/s for input and outlet, respectively. The controlled input liquid flow rate q_{in} is given by:

$$q_{in} = Q_{in} \cdot \sin(\varphi(t)) \quad \varphi(t) \in [0, \pi/2] \tag{14}$$

The output liquid flow rate q_{out} (that equals Q_{out} since no control is applied) is defined as:

$$q_{out} = a_{out}\sqrt{2gh(t)} \tag{15}$$

where a_{out} is the surface area of the outlet and g is the gravitational constant.

The output variable h, which is the level of the liquid, is calculated as:

$$h(t) = h(0) + \frac{1}{A}\int_0^t (q_{in}(\tau) - q_{out}(\tau))d\tau \tag{16}$$

where A is the surface area of the tank.

The numerical values used in this paper are: $A = 1\,m^2$, $a_{out} = 0.01\,m^2$, $Q_{in} = 0.12\,m^3/s$ and $h(0) = 0$.

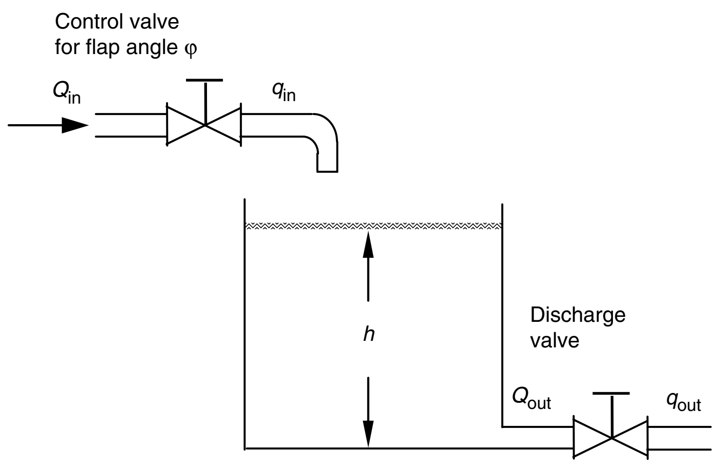

Figure 5 A simple liquid-level system

6. Non-linear model simulations

A number of simulation studies have been carried out in the plant described in the previous section. The proposed controller types have been investigated for two different cases. In case 1, the grey predictor is considered with a fixed step size, and with a variable step size in case 2.

6.1 Case 1: Grey predictor with a fixed step size

The numerical values used in this paper are K=1 and $K_d = 0.1$. Figure 6 shows the response of the system to a PIDFC and a conventional PID controller. As can be seen, the PIDFC has a better capability, when compared with a conventional PID controller in controlling such a non-linear system.

Figure 7 compares the unit step response of the system with a conventional PIDFC and a grey controller with different step sizes, ie, H in Equation (12). As can be seen, when the step size of the grey controller is large, it will cause overcompensation, resulting in a slow system response. Conversely, a smaller step size will make the system respond faster but cause larger overshoots (Wang, 1998). The response with $H = 20$ is better than the one obtained with the fuzzy PID-type controller. Further simulations, shown in Figures 8 and 9, are carried out with this value of H to

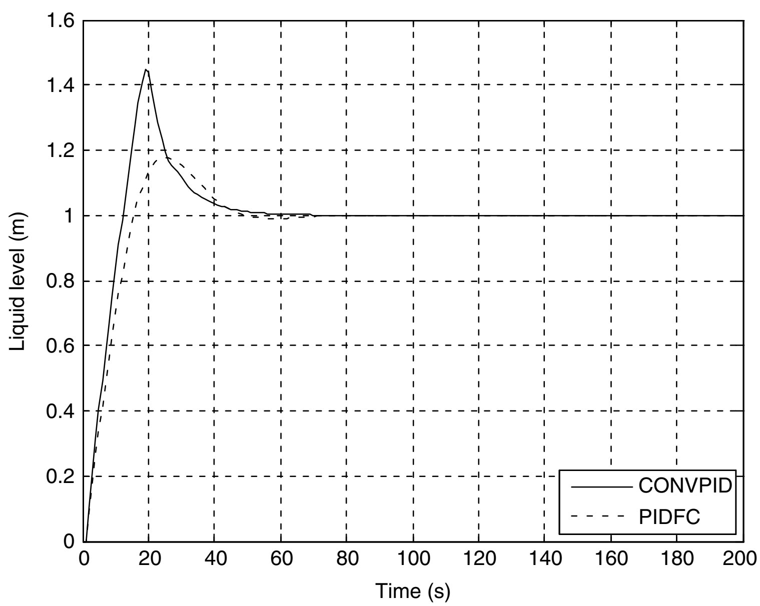

Figure 6 Step responses of the system to PIDFC and conventional PID controller ($\beta = 0.5$, $\alpha = 5$)

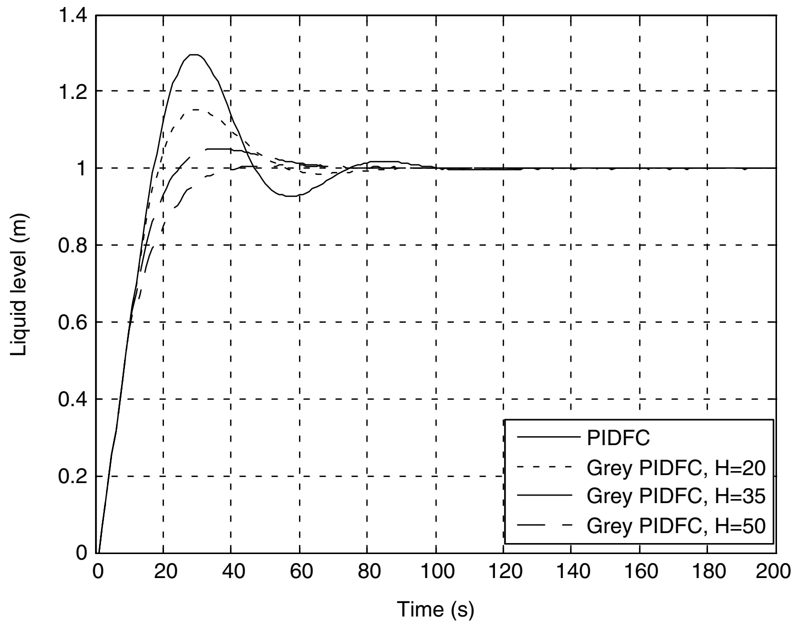

Figure 7 Step responses of conventional and Grey PID-type fuzzy controllers with different step sizes ($\beta = 0.5$, $\alpha = 3$)

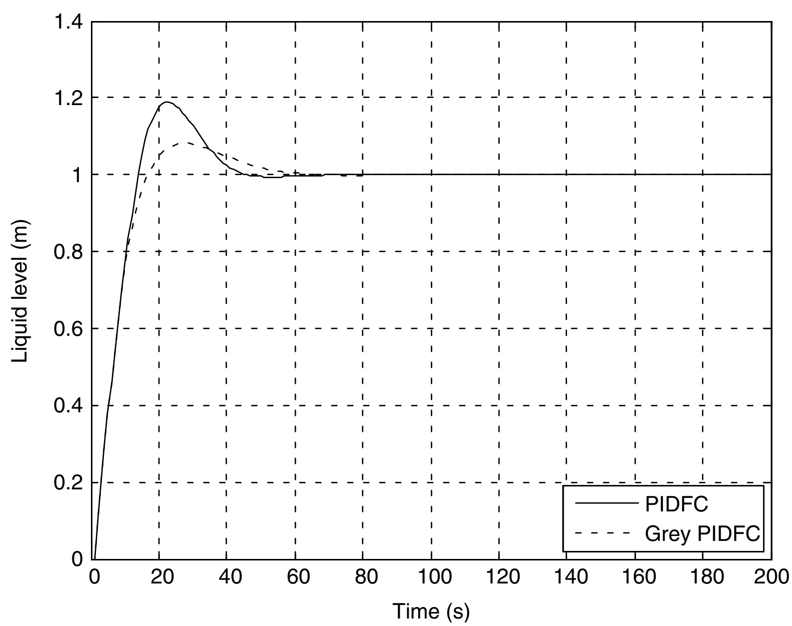

Figure 8 Grey PID-type fuzzy controller structure with different coefficients ($\beta = 0.6$, $\alpha = 6$, H $= 20$)

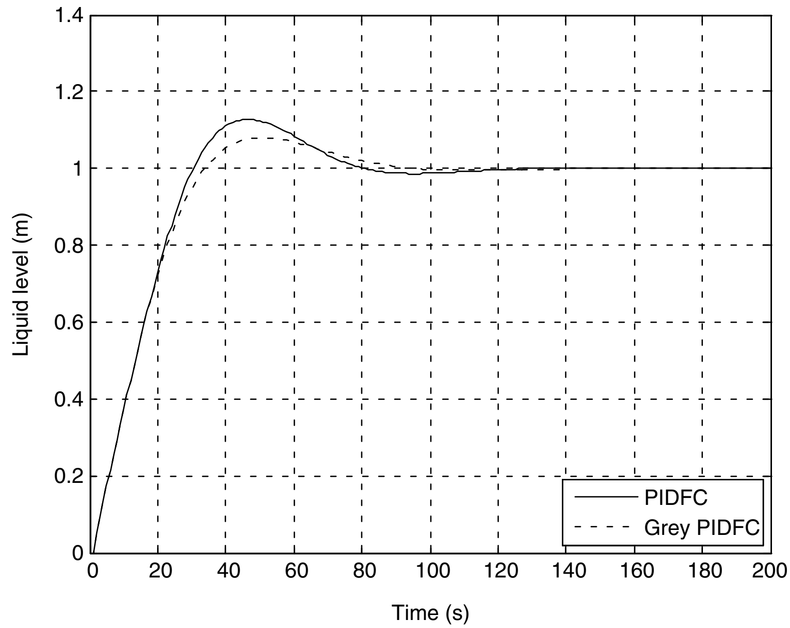

Figure 9 Grey PID-type fuzzy controller structure with different coefficients ($\beta = 0.2$, $\alpha = 2.4$, $\mathrm{H} = 20$)

determine the best parameters of the grey controller. The response shown in Figure 9 has a fast rise time and reasonable overshoot.

Figure 10 shows the unit step response of the system to grey PIDFC with the band-limited white noise at the output measurement. It is seen that the grey PIDFC with grey prediction shows more robust characteristics when compared with a conventional PIDFC.

6.2 Case 2: Grey predictor with a variable step size

In this section, computer-simulated dynamic responses are performed on the non-linear liquid level system; however, a grey predictor with a variable step size is investigated in this time.

Figure 11 shows the step responses of the system to a PIDFC, a grey PIDFC with a fixed H and a grey PIDFC with variable step size. With the grey PIDFC using a variable step size, the system has a fast rise time and a reasonable overshoot. However, a switching characteristic can be seen on the response of the grey PIDFC with variable step size.

Figure 12 shows the unit step responses of the system to a grey PIDFC with a fixed step size and a grey PIDFC with variable step size with the band-limited white noise at the output measurement. The noise power, which is the height of the power spectral

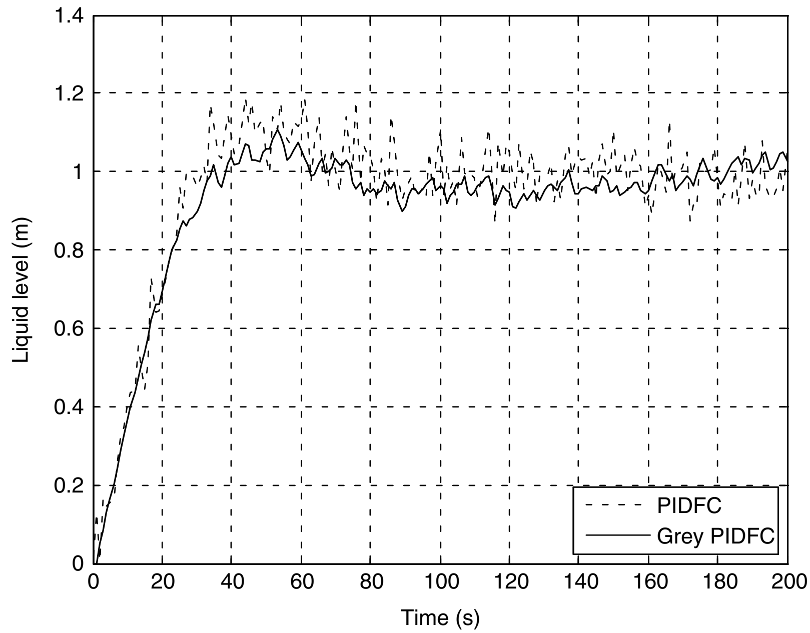

Figure 10 Grey PID-type fuzzy controller with band-limited white noise at the output measurement ($\beta = 0.2$, $\alpha = 2.4$, $H = 20$)

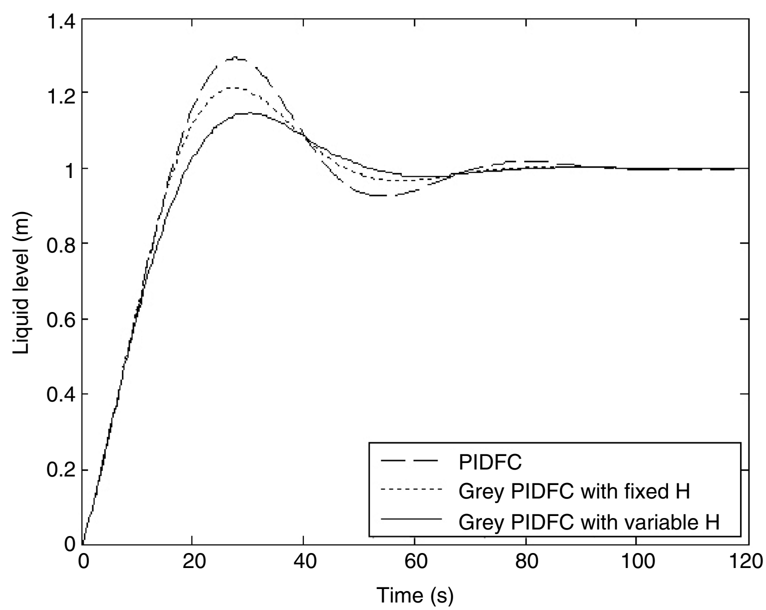

Figure 11 Step responses of the system to a grey PIDFC with a fixed $H = 20$ and a grey PIDFC with variable H ($\alpha = 3.2$, $\beta = 0.6$)

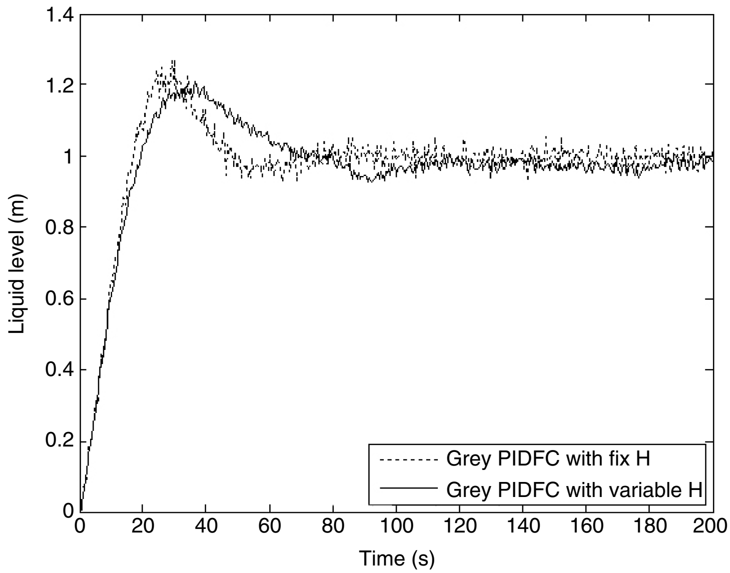

Figure 12 Step responses of the system to a grey PIDFC with a fixed $H = 20$ and a grey PIDFC with variable H when there is white noise at the output measurement ($\alpha = 3.2$, $\beta = 0.6$)

density of the white noise, is equal to 0.0002. The correlation time of the noise is equal to 0.4 s. Although the response of the conventional grey controller is acceptable, the grey predictor with variable step size is better under noisy conditions. This indicates that adaptive grey predictive controllers are more robust in real-time systems that are subject to noise from both inside and outside of the system.

Figures 13 and 14 show the unit step responses of the non-linear liquid level system to a grey PIDFC with a fixed step size and a grey PIDFC with variable step size when the surface area of the outlet a_{out} is reduced to its 0.2 times its normal value in the 100th second and reduced to zero between the 100th and 110th seconds, respectively.

7. Conclusions

In real life, the mathematical model of a physical system cannot be defined exactly; there are always some uncertainties. A control method that has the ability to handle this difficulty would very much be welcomed. In this paper, it is shown that a grey prediction approach is an efficient way of controlling highly non-linear, uncertain systems. The controller described is a combination of a grey prediction approach with a PID-type fuzzy controller. The simulation results presented indicate that the grey prediction model can forecast the future outputs of a grey system to be used to overcome the drawbacks of delay-control methodology. The simulation results also show that the proposed method not only reduces the overshoot and the rise time but

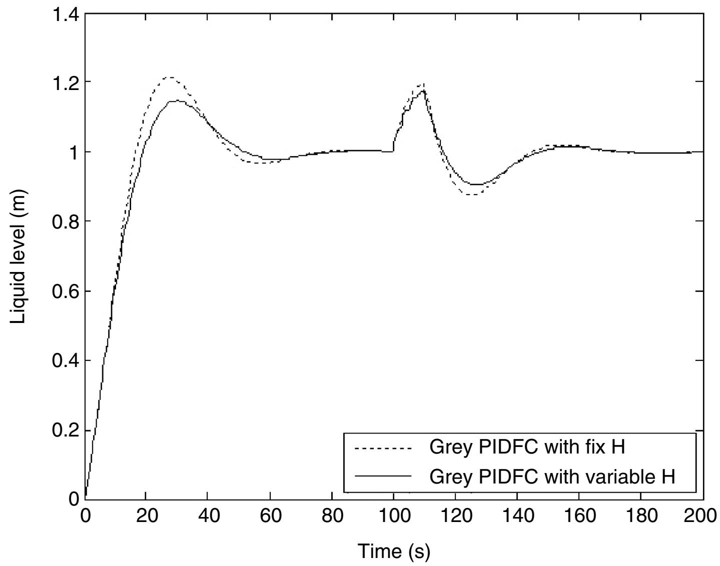

Figure 13 Step responses of the system to a grey PIDFC with a fixed $H = 20$ and a grey PIDFC with variable H when the surface area of the outlet a_{out} is reduced to 0.2 times its normal value ($\alpha = 3.2$, $\beta = 0.6$)

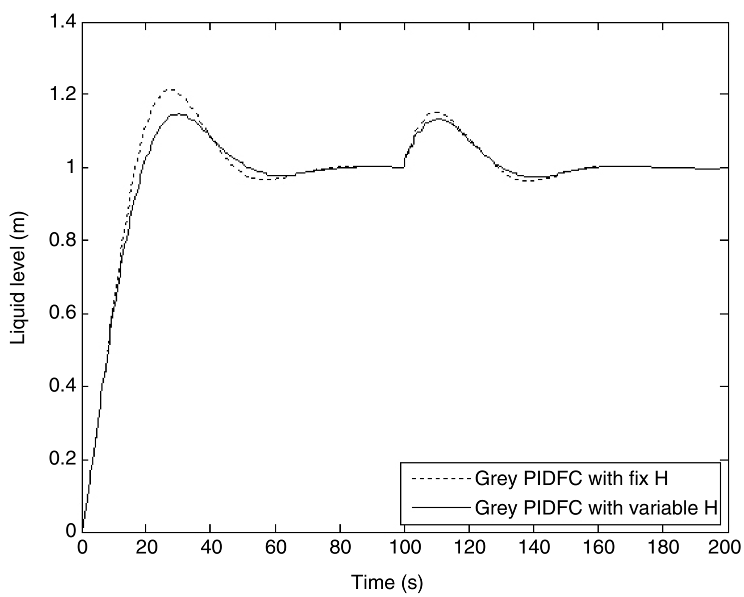

Figure 14 Step responses of the system to a grey PIDFC with a fixed $H = 20$ and a grey PIDFC with variable H when the surface area of the outlet a_{out} is reduced to zero for 10 s ($\alpha = 3.2$, $\beta = 0.6$)

also maintains a better disturbance rejection. Noise that exists in various stages of the system is an additional problem. The proposed adaptive grey PIDFC has the ability to handle these difficulties.

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